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A Systematic Evaluation of Extensions for the Shared Customer Collaboration Vehicle Routing Problem

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ABSTRACT

This paper takes up a recently published model for collaborative vehicle routing, where shared customers expect shipments from more than one carrier. It is the purpose of the model to decide whether the demands of such a customer are served through individual visits of the involved carriers or through a single visit of one of these carriers. We take up this model and extend it here in three ways. First, a cost restriction is included to make sure that no carrier is worse off in the collaboration solution compared to its isolated route planning. Second, we add time windows that play a major role in service-oriented distribution systems. Third, we include inter-depot freight transfers that are needed if a shared customer receives the goods requested at various carriers through a single visit of one of the carriers. Through our systematic consideration of these three extensions, we can conduct comparative experiments that assess the impact of each such extension on the cost savings that are achieved through the collaboration.

KEYWORDS: carrier collaboration · vehicle routing · shared customers · model extensions

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1. INTRODUCTION

With steady population growth and the increasing trend toward e-commerce, traffic in cities has strongly ascended in recent years. The results are congestion, noise, accidents, air pollution, and a related loss in the quality of life of urban population. As Crainic et al. [1] stated, freight transportation contributes significantly to this. But especially in cities where several carriers operate in the same area, last-mile logistics is very competitive. Rising energy prices, environmental regulations, and other restrictions are further aggravating the competitiveness in this market. Since many of the daily tours of competing carriers overlap due to the proximity of their customer base and because the size of the individual transport orders is rather small, the possibility of collaboration has gained increasing attention in this environment. Thereby, one can assume that if a customer is located nearby a route already driven by another carrier who still has free vehicle capacity, the latter will be able to serve this customer more cost-efficiently. This is even more the case if a customer expects shipments from more than one carrier.

In this context, our paper deals with horizontal collaboration where carrier companies are working together such that customers who demand goods from more than one carrier are shared and can be served completely through one of the partners. While most papers in this research area use collaboration strategies in which carriers or forwarders offer all their customers to the collaboration, Fernández et al. [2] developed a model in which both, carriers and customers have influence on which customers will be delivered by which carrier. This particular model, the Shared Customer Collaboration Vehicle Routing Problem (SCCVRP) will be taken up here and supplemented by various relevant extensions.

Our paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we present the SCCVRP model as proposed by Fernández et al. [2] together with a series of own computational results that are used for a comparison with various studies from the field of collaborative transport management and for a subsequent assessment of the impact of the considered problem extensions. Section 4 investigates three extensions to the SCCVRP with regard to adaptations of the model and corresponding computational analyses. The paper is concluded in Section 5.

2. LITERATURE REVIEW

Collaborations in the logistics industry have been studied numerously in recent years. Extensive literature reviews are provided by Gansterer and Hartl [3] and Pan et al. [4] among others. While these refer to diverse areas of application, the survey of Cleophas et al. [5] examines the possibilities of collaboration in urban logistics, including contributions on existing projects, urban consolidation centers, use of public infrastructure, profit distribution, and the composition of collaboration networks.

An early study on the topic of collaboration by means of order sharing is Cruijssen and Salomon [6]. They simulated a centrally organized reallocation of orders among two carriers and identified by experiment a cost reduction potential of 12.3 % on average. In a subsequent sensitivity analysis, they found that the cost advantage of collaboration decreases if the number of orders in the collaboration network increases. This is due to the fact that an increase in the total number of orders also increases the number of orders to be allocated to the individual carriers. This, in turn, means that already the routes planned in isolation by each of the carriers themselves utilize the capacities of their vehicles and, thus, are cost-effective even without collaboration. Furthermore, the cost advantages through collaboration are higher if the collaboration consists of many small carriers rather than a few large ones. With a range of 10 % to 20 %, Krajewska et al. [7] find somewhat greater cost savings potentials in a real world case study of various carriers that act as autonomous profit centers of a freight forwarder all over Germany. They also use a centrally organized route planning for the entire set of orders of the collaboration partners, where the underlying problem is a pick-up and delivery routing problem with time windows. If several planning periods are considered and penalty costs arise for too early deliveries, further savings can be achieved through collaboration, as was shown by Manier et al. [8].

In studies about capacity sharing, where free capacities on already existing routes can be used by collaboration partners, many influencing factors can be found that threaten the cost savings or profit gains from collaboration. For example, Hernández et al. [9] showed in a dynamic route planning model that benefits from collaboration diminish if inventory holding costs arise due to an intermediate storage of goods before they are moved on by a collaboration partner. Similar results are obtained by Joydeep et al. [10], who conduct simulations to show that costs for moving goods to a partner's depot has a large impact on collaboration gains. Nadarajah and Bookbinder [11] even showed that the shape of the customer service area (e.g., a city) has an impact on the savings of mileage traveled achieved in a collaboration.

A crucial factor of the stability of a collaboration is the distribution of profits and cost among the partners, which is why this topic is addressed in the literature too. Although carriers have no interest in sharing orders if there is no individual benefit in doing so, it is quite common that some collaboration partners have to accept higher costs to make the whole collaboration better off. One possibility to deal with this is to share orders within an auction-based environment such that carriers can pick (bundles of) orders that fit into their own route plans, see, for example Berger and Bierwirth [12] or Dai and Chen [13]. Since centralized route planning offers higher cost savings for the collaboration as a whole, an alternative is to compensate those carriers that face higher costs ex post. In this context, Krajewska et al. [7] provide a game-theoretic approach and find by means of Shapley Value a suitable cost allocation that reflects the individual contribution of each carrier to the success of all possible collaboration options. However, there are also counter-effects. For example, the use of the Shapley Value can even reduce satisfaction if individual carriers bring different numbers of customers and vehicles into the collaboration [14]. The chosen method of profit allocation can even have an impact on the strategic behaviour of collaboration partners and, thus, on the height of the generated collaboration profit, see e.g. Dahlberg et al. [15] and Defryn et al. [16].

Since customers may have an aversion against third-party delivery, or carriers may be unwilling to share all their customers, Fernández et al. [2] developed a model that distinguishes between customers that may be served exclusively by one carrier and customers that may be shared in the collaboration network. Their model is presented in detail in the next section. It serves as a basis of various extensions that we consider in our paper to make the SCCVRP more realistic. Some of these have already been proposed by Fernández et al. [2], but were elaborated there only briefly. Since we extend the basic model systematically for all three considered extensions, we open up the opportunity to conduct comparative computational studies that quantify the effects of each of these extensions.

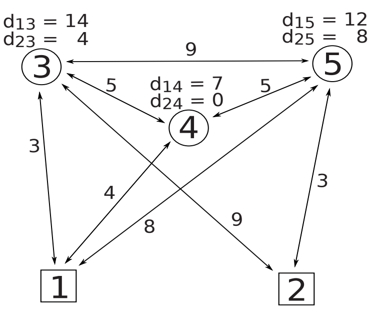
3. THE SHARED CUSTOMER COLLABORATION VEHICLE ROUTING PROBLEM

3.1. Problem Description and Model Formulation

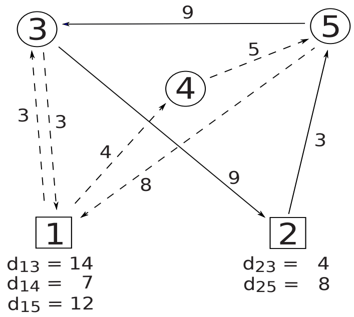
We consider a setting where a given set of customers have to be served by a set of carriers. Each carrier has one depot from which shipments are delivered to customers using the carrier's own vehicles. It is assumed that so-called *shared customers* demand goods from more than one carrier. In order to save costs, the carriers agree to collaborate in a sense of sharing orders within a centralized vehicle routing. The task is then to decide about vehicle routes for serving all customers, where shared customers may be served by a single carrier to reduce overall travel cost of the collaborating carriers. The following notation is used for modelling the SCCVRP. Let $C = \{1, \ldots, m\}$ be a set of m carriers and $N = \{m + 1, \ldots, m + n\}$ be a set of n customers located in a given area. For simplicity, we assume that each carrier has exactly one depot, so that $r \in C$ can represent both a carrier and its depot. With $V = C \cup N$ and $A = \{(i, j) : i, j \in V, i \neq j\}$, $G = (V, A)$ describes a complete directed network, where V represents the nodes and A is the set of directed edges that connect nodes $i, j \in V$. Each edge $(i, j) \in A$ is assigned a weight c_{ij} reflecting the travel cost between the respective nodes. Let the quantity of goods demanded by a customer $i \in N$ at carrier $r \in C$ be denoted by $d_{ir} \geq 0$. Customers with strictly positive demand at carrier $r \in C$ form this carrier's customer base $N_r = \{i | i \in N : d_{ir} > 0\}$. Similarly, for each customer $i \in N$, the set $C_i = \{r | r \in C : d_{ir} > 0\}$ describes those carriers at which this customer ordered goods. We assume that a customer can only be served by its relevant carriers C_i. In turn, we can denote by $V_r = N_r \cup \{r\}$ the subset of nodes and by $A_r = \{(i, j) : i, j \in V_r, i \neq j\}$ the subset of edges being relevant for carrier $r \in C$. Finally, it is assumed that a sufficient number of vehicles with homogeneous capacity Q is stationed at each depot and that all ordered goods are available in all depots. This last assumption will be dropped later in one of the investigated model extensions.

A small example is given in Figure 1. Figure 1(a) shows the problem instance with two carrier depots $C = \{1, 2\}$ and three customers $N = \{3, 4, 5\}$. The set of connections weighted by travel costs and the customers' demands of each of the two carriers are shown too. Sufficient vehicles with a capacity of $Q = 25$ are available at each depot. The cost-minimizing routes in case without collaboration are shown in Figure 1(b), where those demands that are served by a particular carrier are listed below the corresponding depot node in the graph. In this solution, the two routes of carrier 1 (dotted lines in Figure 1(b)) lead to total cost of 23 and the single route of carrier 2 (solid lines) has cost of 21. It can be seen that customers 3 and 5 are visited by both carriers as they demand shipments from both carriers each. In contrast, if the carriers cooperate, it is sufficient that each customer is visited by only one carrier, see Figure 1(c). Here, customer 3's demand is entirely satisfied by carrier 1, customer 5's demand is entirely satisfied by carrier 2, and the total cost of the solution with collaboration is only 18. The total cost of collaboration has thus decreased by about 60 % compared to the solution without collaboration. Note that if a customer or carrier insists, it is possible not to split a customer even if there are demands from several carriers. In this case, the affected customer node is duplicated and the demands are allocated accordingly.

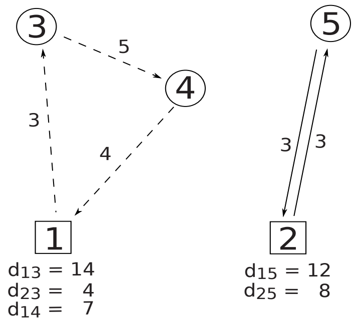
According to Fernández et al. [2], a mathematical model of the SCCVRP can be formulated using the following decision variables. For each carrier $r \in C$ and each edge $(i, j) \in A_r$, a binary routing-variable x^r_{ij} takes value 1 if carrier r traverses edge (i, j) by any of its vehicles, 0 otherwise. To allocate the customer demands to the carriers, a binary variable z_{irs} is used for each customer $i \in N$ and carrier $r, s \in C_i$. It takes value 1, if the demand from customer $i \in N$ at carrier $r \in C_i$ is fulfilled by carrier $s \in C_i$, 0 otherwise. Finally, to guarantee that loads are delivered to the correct customer and to ensure that vehicle capacities will not be exceeded, a continuous load-variable



(a) Problem instance



(b) Optimal solution without collaboration



(c) Optimal solution with collaboration

Figure 1: Example

l_{ij}^{rh} is introduced that calculates the load carried by carrier $r \in C$ to customer $h \in N_r$ traversing edge $(i, j) \in A_r$.

Then, the formulation of the SCCVRP is as follows:

$$\min \sum_{r \in C} \sum_{(i,j) \in A_r} c_{ij} x_{ij}^r \tag{1}$$

$$\text{s.t.} \sum_{s \in C_i} z_{irs} = 1 \qquad i \in N,\ r \in C_i \tag{2}$$

$$\sum_{j \in V_r : j \neq i} x_{ij}^r - \sum_{j \in V_r : j \neq i} x_{ji}^r = 0 \qquad i \in N,\ r \in C_i \tag{3}$$

$$\sum_{j \in V_s : j \neq i} x_{ij}^s \geq z_{irs} \qquad i \in N,\ r, s \in C_i \tag{4}$$

$$\sum_{j \in N_r} l_{rj}^{rh} = \sum_{s \in C_h} d_{hs} z_{hsr} \qquad r \in C,\ h \in N_r \tag{5}$$

$$\sum_{j \in V_r : j \neq i} l_{ij}^{rh} - \sum_{j \in V_r : j \neq i} l_{ji}^{rh} = \begin{cases} 0 & h \neq i \\ -\sum_{s \in C_i} d_{is} z_{isr} & h = i \end{cases} \qquad r \in C,\ i, h \in N_r \tag{6}$$

$$\sum_{h \in N_r} l_{ij}^{rh} \leq Q x_{ij}^r \qquad r \in C,\ (i, j) \in A_r \tag{7}$$

$$x_{ij}^r \in \{0, 1\} \qquad r \in C,\ (i, j) \in A_r \tag{8}$$

$$z_{irs} \in \{0, 1\} \qquad i \in N,\ r, s \in C_i \tag{9}$$

$$l_{ij}^{rh} \geq 0 \qquad r \in C,\ h \in N_r,\ (i, j) \in A_r \tag{10}$$

Objective function (1) minimizes the total routing cost of the collaborating carriers. Constraints (2) guarantee that each customer is served by one of the carriers at which it placed a demand. Note that a carrier supplies its customer itself if $r = s$ whereas this customer will be shared to another carrier if $r \neq s$. Constraints (3) ensure that each customer node, once entered, is exited and (4) establishes the relationship between the variables x and z. Constraints (5) to (7) regulate the distribution of the load. Here, (5) states that the sum of all load quantities leaving a depot r must be identical to the sum of demands satisfied from there. According to constraints (6), loads designated for a customer h but arriving at some customer node $i \neq h$ must stay on the vehicle. However, if $h = i$, the load destined for that node will completely be unloaded through constraints (6). Constraints (5) and (6) thus guarantee that loads that have once left the depot are also delivered. At the same time, they ensure that subtours are avoided, since a load that has already been delivered cannot be picked up again at a node visited later. Constraints (7) establish the connection between the x and l variables and ensure that the vehicle capacity is not exceeded. Constraints (8) to (10) specify the domains of the decision variables.

3.2. Computational experiments

We have generated 100 test instances for a computational evaluation of the SCCVRP. All of them are based on a square area of 50x50 distance units within which the customer nodes were randomly distributed. For a first group of 40 instances, $m = 2$ carriers were randomly positioned (Prefix R). Ten of each of these instances contain 10, 15, 20, and 25 customer nodes. The probability of belonging to the shared customers is denoted ϱ and set to 25 % for every five of the instances and to 50 % for the other five instances. The remaining customers were randomly assigned to one of the two carriers. The 40 R-instances where then duplicated with carriers being set to fixed coordinates (1,1) and (50,50). These instances receive prefix C. Finally, 20 instances with $m = 3$ randomly

Table 1: Cost change ΔC (%) of a collaboration of $m = 2$ carriers in the SCCVRP

n/ϱ:	10/25	10/50	15/25	15/50	20/25	20/50	25/25	25/50
R-instances:	-20.18	-27.11	-12.22	-23.43	-13.71	-25.28	-15.40	-20.83
C-instances:	-13.30	-18.24	-14.13	-23.25	-17.17	-23.48	-14.47	-21.72

positioned carriers were generated and given the prefix R too. Ten of each of these instances contain 15 and 20 customer nodes randomly distributed on the map. The probability ϱ of being a shared customer varies from 40 % to 93 % in these instances. The capacity of the vehicles was set to 100 for all instances and the demand of each customer $i \in N$ was randomly drawn from the interval [5, 20], using only integer values. We want to note that the first two groups of our instances are similar to those used by Fernández et al. [2]. However, the probability of belonging to the shared customers was fixed to 25 % for their main set of test instances whereas we vary this parameter.

For the computations, the SCCVRP model has been implemented in CPLEX version 12.10.0 and solved on a core i5 with 4 x 2.9 GHZ and 16 GB RAM. The runtime was limited to 2 hours for each instance. If this time limit was reached, the best feasible solution found so far was taken.

A summary of the results of our first two groups of instances with $m = 2$ carriers can be seen in Table 1. This table reports the relative change in cost ΔC that is observed, if carriers collaborate by exchanging customers compared to the solution where each carrier serves its own customers exclusively. The reported cost change ΔC is the average value of five instances, grouped by number of customers n and their probability of being shared ϱ. It can be seen that an increasing number of customers does not necessarily lead to higher relative cost savings. For the R-instances, the relative cost saving for instances with 10 customers are even better than those with any larger number of customers under a same probability ϱ. This finding is in line with Fernández et al. [2] but does not confirm the observation made by Cruijssen and Salomon [6] that cost advantages decrease with an increasing number of orders in the collaboration network. However, the absolute number of ordering customers is at most 25 in our instances, so that it cannot be ruled out that this effect does occur with a further increase in the number of customers. Nevertheless, it can be observed that savings increase, if the percentage of shared customers increases. On average, these are 15.07 % with $\varrho = 25$, and 22.92 % with $\varrho = 50$. This suggests that it is not the number of customers, but the proportion of shared customers that has a positive effect on collaboration success, which is an expected outcome. The explanation for this is as follows. As the number of customers increases, the number of shared customers increases, but so does the number of customers to be supplied individually. Resulting trips then lead across the whole geographic area. If the potential of sharing increases through higher values of ϱ, both, the probability that more distant customers will be shared and the probability that a shared customer will be near a non-shared customer increases. Routes can then be generated more efficiently.

A closer look at the results shows that the total cost of the collaboration, in absolute terms, is lower in case of randomly distributed carriers. This effect could already be observed by Fernández et al. [2] and may be explained by the fact that a carrier, positioned in one corner of the area and having a non-shared customer on the opposite corner, has to cover a comparatively long distance that partially offsets the advantages arising from collaboration.

If $m = 3$ carriers are involved, the cost change ΔC as an average of 10 instances with 15 and 20 customers each is 34.09 % and 38.56 %, respectively. Compared to the instances with $m = 2$ carriers, a larger relative savings potential is observed, which correspond to findings reported by Cruijssen and Salomon [6], see Section 2. However, since the probability of being a shared customer is comparatively higher in the 3-carrier-instances, the higher saving cannot be attributed solely to the increased number of carriers.

From evaluating the number of vehicles being used in the obtained solutions, we observe that the collaboration saves one vehicle in around 40 % of the instances. However, with $m = 2$ carriers involved, in fourteen instances the collaboration leads to one of the partners needing one or two more vehicles while the other partner saved the equivalent number of vehicles. If three carriers are involved, such a shift results for almost half of the evaluated instances. We even observed two solutions where one of the carriers no longer carried out any orders itself. In this case, individual carriers will likely choose not to share some of their customers to cover their fixed costs. However, if only the operational costs are considered, as is the case here, the distribution of the cost saving among the collaborating carriers can differ drastically and it can well happen that one carrier has to carry higher cost in the SCCVRP solution compared to the optimal solution without collaboration. In the instances with two carriers, the individual costs for one carrier increased in the solutions to twelve instances. Under $m = 3$ carriers, we observed this for two instances. Since such effects threaten the acceptance of a collaboration, we investigate subsequently an extension of the SCCVRP that guarantees win-win-solutions for the involved carriers.

4. EXTENSIONS OF THE SCCVRP

The basic version of the SCCVRP as presented in the previous section can be extended in various ways to incorporate further features of practical relevance. In the following, we present, formulate, and test three such extensions. The first extension adapts the SCCVRP such that none of the carriers suffers from a cost increase in the collaborative solution. The second extension is to include time windows into the SCCVRP, which respects that route planning in service-driven applications often involves delivery time windows that are agreed with customers. A last extension incorporates freight transfers between depots for those customer orders that are shared among carriers.

4.1. The SCCVRP with restricted cost

To raise the interest of carriers for a participation in a collaboration, a profit threshold could be established such that no carrier faces a reduced profit in the collaborative solution compared to the solution without collaboration (see e.g. [17]). As the SCCVRP focuses on cost rather than profits, we introduce a variant of the SCCVRP in which no carrier faces higher cost in the collaborative solution compared to acting in isolation. This extension is modelled as follows. Let C_r denote the (precomputed) costs incurred by a carrier $r \in C$ in the optimal isolated routing without collaboration. Similarly, let C_r^{coll} denote the costs of carrier r in the collaborative SCCVRP-solution. Then, the SCCVRP-model (1) to (10) is augmented by the following constraints

$$\mathsf{C}_r^{coll} = \sum_{(i,j)\in A_r} c_{ij}x_{ij}^r \qquad r \in C, \qquad (11)$$

$$\mathsf{C}_r^{coll} \leq \mathsf{C}_r \qquad r \in C, \qquad (12)$$

where constraints (11) compute the cost of carrier r in the solution with collaboration and constraints (12) restrict this cost to be at most C_r.

An investigation of the original SCCVRP solutions from Section 3.2 has revealed that constraints (12) are violated for at least one of the carriers for a total of fourteen instances (four of the R-instances with 2 carriers, eight of the C-instances with 2 carriers, and two of the R-instances with 3 carriers). We then resolved these instances using the SCCVRP-model with restricted cost. Table 2 shows the results for each of these fourteen instances. It reports the individual cost C_r^{coll} of each carrier r in the collaborative solution, the total cost of the solution C^{coll}, and the relative change in cost $\Delta\mathsf{C}$ that is achieved in the collaborative solution compared to the non-collaborative solution. This information is shown in Table 2 for the solutions of the basic SCCVRP and the solutions of the SCCVRP with a cost restriction. The results reveal that adding a cost restriction for all carriers reduces the savings $\Delta\mathsf{C}$ that can be achieved through collaboration. Anyhow, substantial collaboration-based cost savings of at least 5 % are still observed in all these instances, and the drop in the cost saving between the SCCVRP with restricted cost and the basic SCCVRP is just a few percent for the majority of the instances.

As an example for the effect of the cost restriction, we take a closer look at instance R_{26}. For this instance, each of the $m = 2$ carriers has cost of about 190 in the solution without collaboration (not shown in the table). From solving the basic SCCVRP, carrier 1 faces costs of 239.71, while carrier 2 faces costs of only 39.28. This involves a drastic reduction of the cost of carrier 2 but, at the same time, a substantial increase of the cost of carrier 1. Using the SCCVRP with cost restriction, the costs are now 174.38 and 114.32, respectively. From this, one of the carriers can reduce its costs by about 42 %, whereas the other realizes a saving of about 8 % compared to the non-collaborative solution. It can be seen that none of the carriers is now worse off in the collaborative solution. With this cost split, a stable collaboration may occur,

Table 2: Results for basic SCCVRP and SCCVRP with restricted cost

	SCCVRP					SCCVRP with restricted cost				
instance	C_1^{coll}	C_2^{coll}	C_3^{coll}	C^{coll}	$\Delta\mathsf{C}(\%)$	C_1^{coll}	C_2^{coll}	C_3^{coll}	C^{coll}	$\Delta\mathsf{C}(\%)$
R_{13}	267.05	136.48		403.54	-17.50	251.25	206.91		458.16	-6.33
R_{26}	239.71	39.28		279.00	-29.98	174.38	114.32		288.70	-25.48
R_{33}	260.19	141.76		401.96	-18.33	192.41	218.41		410.83	-16.53
R_{38}	136.74	303.57		440.32	-15.19	222.84	220.70		443.54	-14.57
C_{01}	152.45	116.99		269.45	-15.98	120.14	184.22		304.37	-5.09
C_{09}	180.77	112.49		293.26	-13.02	137.91	168.65		306.56	-9.07
C_{13}	226.68	141.87		368.57	-21.64	193.36	217.37		410.73	-12.67
C_{17}	140.77	205.92		346.69	-17.53	209.06	144.73		353.80	-15.84
C_{20}	252.57	149.37		401.95	-17.57	190.08	233.04		423.12	-13.23
C_{23}	148.88	248.42		397.31	-12.85	189.09	232.27		421.37	-7.58
C_{36}	176.44	247.00		423.45	-20.13	194.72	236.78		431.51	-18.61
C_{40}	221.42	259.48		480.91	-18.42	291.97	199.75		491.72	-16.58
R_{48}	0.00	89.29	212.92	302.21	-32.16	92.49	89.29	147.43	329.21	-26.10
R_{58}	94.70	40.24	237.22	372.18	-29.29	150.64	40.24	187.22	378.11	-28.16

but still strongly in favor of carrier 2. It, thus, remains somewhat open whether this distribution of costs will be perceived as fair by the carriers.

For a brief comparison, we compute the cost split as it would follow from the Shapley Value. This value derives the cost of carrier $r \in C$ by

$$\mathsf{C}_r^{Shapley} = \sum_{S \subseteq C \setminus \{r\}} \frac{|S|!(|C| - |S| - 1)!}{|C|} \big(\mathsf{C}(S \cup \{r\}) - \mathsf{C}(S)\big) \qquad \forall r \in C, \tag{13}$$

where S iterates over all sub-coalitions being possible without the currently inspected carrier r ($S = \emptyset$ is allowed too) and $\mathsf{C}(\cdot)$ denotes the costs incurred in the respective composition of collaboration. From this calculation, carrier 1 would get allocated costs of 135.92 and carrier 2 would get allocated cost of 143.08. The allocation of costs is therefore more balanced and, thus, in line with the cost ratio of the carriers before the collaboration.

4.2. The SCCVRP with Time Windows (SCCVRP-TW)

In many applications for collaborative route planning, customers can only be visited at certain times. This may be the case for business customers that have limited opening hours, for attended home-deliveries where people need to be at home for receiving shipments, or for temporary entry or parking permits in certain areas of a city center. Also under such time windows, collaboration gains can be achieved, as was already discussed in Section 2. However, so far no studies exist on the SCCVRP where customers can be shared but carriers have to respect time windows. We therefore extend the SCCVRP by time windows constraints, leading to the SCCVRP-TW. For this, we assume that the service start time at customer $i \in N$ may only be within a fixed and given time window $[a_i, b_i]$ with $a_i < b_i$ and $a_i, b_i \in \mathbb{R}$, where waiting of a vehicle is allowed if it arrives before time a_i. Furthermore, let t_{ij} be the travel time along edge $(i, j) \in A$. Finally, we define decision variable y_{ri} as the service start time of carrier $r \in C$ at customer $i \in N_r$. This variable allows different carriers to deliver to a same customer at different times as long as all these times are within the time window. The SCCVRP-TW is then obtained from SCCVRP-model (1)-(10) through adding the constraints

$$a_i \leq y_{ri} \leq b_i \qquad r \in C,\ i \in N_r, \tag{14}$$

$$y_{rj} \geq y_{ri} + t_{ij} - M(1 - x_{ij}^r) \qquad r \in C,\ i, j \in N_r : i \neq j. \tag{15}$$

Here, constraints (14) guarantee that the arrival time of carrier $r \in C$ at customer $i \in N_r$ is within the specified time window. Constraints (15) ensure that at least t_{ij} time units must elapse between visiting two nodes i and j if they are visited consecutively on a route. Note that if customer i is not served by carrier r in the considered solution, y_{ri} remains undefined and takes an arbitrary value within the time window.

For experimenting with the SCCVRP-TW, we extended our SCCVRP instances by time windows. The starting time a_i of the time window of customer i is randomly chosen between 10am and 3pm. The width of a time window $(b_i - a_i)$ is chosen randomly between one to three hours. We furthermore assume that drivers may work for a duration of 10 hours per day which is why we set $M = 10$ in constraints (15). For the calculation of travel times t_{ij}, the transit speed for each edge was set randomly to 40 or 44.4 km/h in order to simulate a different traffic load.

Table 3 summarizes the results obtained for the SCCVRP-TW. Like in Table 1, we show the total relative change in costs $\Delta\mathsf{C}$ in comparison with no collaboration as an average of five instances. In general, Table 3 reveals that the SCCVRP-TW opens up substantial cost saving potentials compared to solutions where carriers do not collaborate. These savings range from 14.35 % to 35.19 % for the various instance sets. From comparing these results with Table 1, we see that the relative savings in the SCCVRP-TW are even higher than in the SCCVRP without time windows. The savings do not necessarily grow with an increasing number n of customers but they clearly grow with a larger percentage ρ of shared customers. More precisely, savings for instances with $\varrho = 25$ range from 14.35 % to 21.69 % whereas instances with $\varrho = 50$ achieve savings between 23.75 % and 35.19 %. The effect is more distinguished than for the original SCCVRP, where the corresponding ranges where 12.22 % to 20.18 % and 18.24 % to 27.11 %, respectively, see Table 1. However, while the relative savings are higher in the SCCVRP-TW, the absolute costs of a solution exceed those of the SCCVRP due to the additional time restrictions that have to be respected. Accordingly, the SCCVRP-TW solutions

Table 3: Cost change $\Delta\mathsf{C}$ (%) of a collaboration of $m = 2$ carriers in SCCVRP-TW

n/ϱ:	10/25	10/50	15/25	15/50	20/25	20/50	25/25	25/50
R-instances:	-21.69	-35.19	-14.35	-27.74	-17.20	-29.40	-18.82	-25.25
C-instances:	-20.06	-30.40	-15.36	-25.11	-15.41	-27.60	-14.93	-23.75

also involve a larger number of vehicles. In a sense, this confirms the results found by Manier et al. [8], namely that costs can be saved the more relaxed delivery time restrictions are. Finally, for instances with $m = 3$ carriers (not shown in Table 3), the savings range from 35.38 % to 45.53 %, which indicates that the more carriers participate in the collaboration, the higher is the benefit in the SCCVRP-TW. As these percentages are once more higher than in the SCCVRP without time windows (Section 3.2), it again confirms that restrictive time windows call for more collaboration.

4.3. The SCCVRP with inter-depot freight transfers

In the basic SCCVRP, it is assumed that all goods delivered by the collaboration partners are available in all depots such that a shared customer can be served from any of the carriers. If this assumption does not hold, goods must be transferred between depots to ensure that a dedicated shipment for a customers is at the right carrier's depot. Fernández et al. [2] present an extension where a transfer of goods between depots takes place if a shared customer is shifted from one carrier to another. They assume unlimited capacity for the transfer vehicle and add a constant fixed cost for each transfer taking place in a solution. In the following, we present a somewhat more realistic version of this extension, where we consider vehicle capacities and decide on the number of vehicle trips being required to transfer cargo from one depot to the other. Furthermore, we use cost rates c_{rs} when assessing these vehicle trips to respect the inter-depot distances. We refer to the resulting variant as the SCCVRP with inter-depot freight transfers.

Our extension is modelled through additional integer decision variables $v_{rs}\ \forall r, s \in C,\ s \neq r$, which represent the number of vehicles that transfer cargo from depot r to depot s. As variables v_{rs} are defined for $s \neq r$, transfers take place in a directed manner. E.g., if carrier 2 takes orders from carrier 1, at least one vehicle travels from depot 1 to depot 2 whereas there is not necessarily a transfer in the opposite direction. With our approach, transfer is thus calculated as a one-way-trip, so that the transfer vehicle does not have to return empty to its depot of origin, but is directly available for another use. The SCCVRP with inter-depot transfers is then obtained as

Objective (16) minimizes the total cost incurred for traveling the edges as before, but now supplemented by the cost incurred for transferring cargo between any two depots. Constraints (17) respect vehicle capacities and ensure that a sufficient number of vehicles is deployed for conducting these transfers. Constraints (18) specify the domains of the added decision variables.

Fernández et al. [2] find in their version with fixed transfer costs, that the savings generated by a collaboration depend strongly on the height of these transfer costs. The results of their calculations thus confirm the results of Joydeep et al. [10], who also showed that collaborations are less profitable the higher such transfer costs are. Our own results confirm this to some extent. When transfer costs are taken into account, the solutions did not always result in a collaboration. Table 4 therefore only shows average results for the number of those instances (no.) in which a solution still involved a collaboration of carriers. In addition to the savings ΔC, we also report by δC^{tr} the ratio of the transfer costs (second term in (16)) to the total cost of the solution. The results show that there are still substantial savings of up to 16.92 % in comparison to the non-collaborative solutions. However, the relative change in total cost ΔC clearly drops compared with the previous experiments. For the R-instances, with exception of the group $n/\varrho = 25/50$, where two instances were not included in the evaluation because no feasible solution was reached within the limited runtime, the savings are substantially higher with $\varrho = 50$ compared to $\varrho = 25$. Interestingly, this comes at only slightly higher values δC^{tr}. This shows that the transfer costs can partially be offset as the number of shared customers increases and that the vehicles that are used for transfers anyways are better utilized if more shared customers are available. As the cost for inter-depot transfers depends on the distance between depots in our proposed model extension, the number of instances where collaboration takes place is lower for the C-instances. For $n = 10$, there are even no more collaborations at all. For the remaining groups, as above, we find a slightly greater chance of collaboration with a larger number of customers, but with relatively small savings. Nevertheless, collaborations arise in every solution with $m = 3$ carriers, with average cost savings of 14.91 % for $n = 15$ customers and 22.26 %

$$\min \sum_{r\in C} \sum_{(i,j)\in A_r} c_{ij} x_{ij}^r + \sum_{r,s\in C: s\neq r} c_{rs} v_{rs} \tag{16}$$

s.t. constraints (2) - (10)

$$\sum_{i\in N_r\cap N_s} d_{ir} \cdot z_{irs} \leq Q \cdot v_{rs} \qquad r, s \in C : s \neq r \tag{17}$$

$$v_{rs} \in \mathbb{N} \qquad r, s \in C,\ s \neq r \tag{18}$$

Table 4: Cost change ΔC (%) and ratio δC^{tr} (%) of transport cost to total cost from a collaboration of $m = 2$ carriers in the SCCVRP with inter-depot freight transfers

	n/ϱ:	10/25	10/50	15/25	15/50	20/25	20/50	25/25	25/50
	no.	5	5	3	5	4	5	5	2
R-instances:	ΔC	-8.95	-12.92	-7.79	-9.74	-6.95	-16.92	-5.55	-5.71
	δC^{tr}	10.83	13.16	9.79	10.38	6.70	7.79	7.60	7.45
	no.	0	0	2	5	1	4	2	5
C-instances:	ΔC	-	-	-4.96	-4.77	-4.47	-4.64	-3.51	-2.75
	δC^{tr}	-	-	8.18	16.45	14.99	17.05	12.34	11.90

for $n = 20$ customers, which, again, is lower compared to the basic SCCVRP.

5. CONCLUSIONS

In this paper, we have taken up the SCCVRP model for collaborative vehicle routing as proposed by Fernández et al. [2] and investigated three relevant extensions for this: The SCCVRP with restricted cost, the SCCVRP with time windows (SCCVRP-TW), and the SCCVRP with inter-depot freight transfers. By considering these three extensions within our paper, we can conduct systematic experiments that assess the impact of each such extension on the collaboration based cost savings in comparison to the original model, compared to a non-collaborative solution, and mutually among the extensions. Experiments on 100 test instances showed that the basic version of the SCCVRP can achieve cost savings of 12.22 % to 38.56 % due to a sharing of customers among the collaborating carriers. Savings increased when the proportion of shared customers increased and decreased when the depots were positioned in the corners of the delivery areas compared to a random positioning. Extending the SCCVRP by a cost restriction ensures that no carrier faces higher cost in the collaborative solution, which, in turn, slightly reduced the savings that are obtainable through customer sharing. In contrast, in case that customers have to be delivered within given time windows, relative cost savings through collaboration even increased up to 45.53 % as was revealed by the computational experiments for the SCCVRP-TW. The final extension addressed freight transfers among the collaborating depots. Contrasting Fernández et al. [2], who included this into their model under the very strong assumption of unlimited vehicle capacity, we proposed here a model extension that explicitly respects vehicle capacity and, thus, decides on the number of vehicle trips being required for inter-depot transfers. As these transfers come at a cost, collaboration is no longer attractive for some of the test instances. For those instances where collaboration still plays a role, the achieved cost savings ranged from 2.75 % to 22.26 %. The model variants presented here thus show that freight carriers can gain cost advantages through collaboration in various settings. Even under costly inter-depot freight transfers, collaborations can be efficient, although savings will then be lower. Models such as the SCCVRP with a cost restriction can help to establish trust and motivate carriers to participate in collaborations. However, additional agreements on the distribution of the resulting costs may be indispensable.

Although this paper achieves a systematic comparison and evaluation of three extensions of the SCCVRP, there is still a number of further extensions that might be considered in future research. Such open extensions are, for example, to investigate the impact of limited fleet sizes or heterogeneous vehicles that differ in their capacities and cost, the distribution of the saved costs among the collaborating partners, or time-dependent travel times that may be of importance especially in the SCCVRP-TW. In addition, since operational costs are often low compared to fixed costs in the transportation sector, further analysis is needed to determine whether our results hold if more realistic cost structures or profits are included in the considerations.

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