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Time-differentiated supply at customer sites: Analysing the service costs

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ABSTRACT

Common in IT equipment support and emergency service operations, time-based service differentiation involves setting different service time windows to respond to different classes of service calls. These time windows can be associated with providing service parts (materials) at customer sites. It is reasonable to expect that the impact of different service time options on service costs is realized for strategic and operational planning. However, the question of how different service time limits impact on service costs has largely been overlooked in the existing literature. The paper focuses on this question and presents an estimate cost model based on a stylized system considering hierarchical and non-hierarchical organizations of service facilities. The impact of varying service time limits and the demand fractions for different service times on inventory, transportation and distribution network setup costs is investigated. The findings show that when service time requirements become stricter, requiring more decentralized distribution, the inventory levels do not increase in all cases, while travelling to reach customers reduces proportionally. The analysis highlights that a non-hierarchical setup of facilities, treating different service time requests in a uniform fashion, can perform better in certain cases.

KEYWORDS Distribution · Inventory · Transportation · Decentralization · Time constrained services

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1. INTRODUCTION

Many multinational manufacturing companies have moved towards becoming service providers. The service provider role of the major manufacturers is prevalent in sectors like Information and Communication Technology (ICT), automotive, and aerospace where the companies offer a range of after-sales services. One of the forms of after-sales services is to provide service parts (also known as spare parts and repair parts) for equipment maintenance and repairs at customer sites. This study is mainly motivated by Service Parts Logistics (SPL) systems of ICT equipment manufacturers and service providers that support provision of service parts under different and short service time commitments to the customer base, which is spread over a large geographical area. Typically in ICT hardware support contracts, depending on the consequences of their equipment downtime, customers determine different time windows (e.g. 2 hours, 4 hours and 8 hours) within which the requested service part(s) should reach their sites. The study is, however, generic in nature and generates insights that can be relevant for cases where the service responses are provided within different short time windows and involve the provision (or consumption) of stock (e.g. emergency infrastructure repairs on the road network).

Differentiation is not a new topic in inventory research. The inventory research relating to differentiation dates back to the 1960s. The problem of multiple demand classes was introduced by Veinott Jr [1], who focused on the question of how much to order and when to replenish within a periodic review system where, in each period, the requests are satisfied in a sequence which is in accordance with the priority of their classes. Since this study, a stream of service-differentiated research has been published, the majority of which is in the context of service parts and relates to fill-rate (service availability) based differentiation through inventory rationing. Time-based service differentiation has received comparatively less attention in inventory and facility location research. There are few studies that consider differentiating customers based on different service (supply) time options that they choose. Kranenburg and van Houtum [2] consider a single facility and multiple customer groups, each having a service level equal to the maximum average waiting time at the warehouse. The facility has a normal and an emergency replenishment option: the emergency replenishment mode is used in case of a stock-out and is quicker and more expensive. Their model seeks to determine the stock level at the facility that minimizes the cost (sum of inventory holding, normal replenishment, and emergency replenishment costs) while meeting the average waiting time target for each customer group. Kranenburg and van Houtum [3] study a similar system as in [2], but they use a stock rationing policy as a means to offer a fill-rate differentiation to the customer groups as well. Kranenburg and van Houtum [4] extend the model in [2] by considering multiple warehouses instead of a single facility system. These studies [2-4] only consider customer waiting times at warehouses, assuming that customers instantly get parts when they are released by a warehouse. The travel distance to reach customers, which can be an important factor when the service times are short (e.g. typically a few hours in IT equipment support services), is not considered. Apparently the travel time can make a considerable difference in the overall service time [5, 6]. Service time constraints imply a demand covering issue within certain distance ranges associated with the maximum service times. The SPL studies that do account for service time constraints and warehouse-customer distances for inventory and facility location decisions, on the other hand, lack the service differentiation aspect [e.g. 7-12].

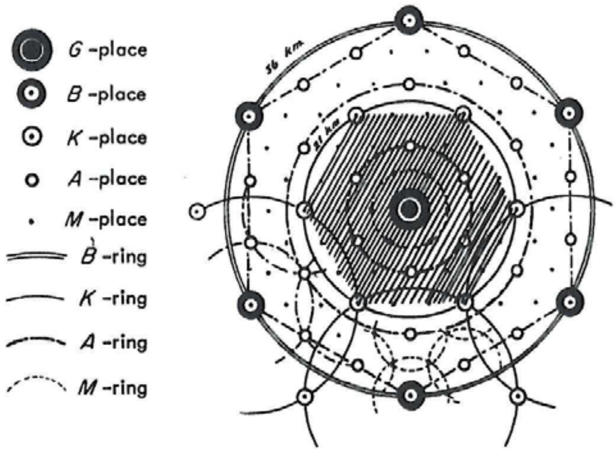
The number of stocking facilities in a distribution network affects its responsiveness and costs. Setting the level of centralization/decentralization in a distribution network is a well-argued strategic decision and has various trade-offs (see [13]). These trade-offs include warehouse-to-customer travelling distances, inventory costs, and responsiveness. A centralized distribution setup results in lower inventory costs as, compared to a decentralized setup, it requires a lower inventory level to attain a particular service level. On the other hand, a decentralized distribution network can result in better responsiveness and lower warehouse-to-customer travelling distance. The early text books referring to stock centralization and decentralization concepts include [14, 15]. Key differences between attributes of centralized and decentralized strategies for after-sales services are identified in [16, 17]. Studies generally assume that a supply in shorter time windows results in higher costs (e.g. [16-18]). However, the exploration into how different service time limits and the demand composition impact on distribution costs has been overlooked in the facility location literature generally and the SPL literature specifically.

Most SPL studies are focused on the optimization of inventory policy parameters. Kennedy et al. [19] discuss unique aspects of service parts inventories and review the related literature. Muckstadt [20] presents details of service parts inventory systems and supply chain algorithms, and provides an extensive bibliography on the subject. A considerable portion of research on service parts inventory management has focused on lateral transhipment or inventory sharing. Paterson et al. [21] provide a comprehensive review of the literature in this area till 2011, while Patriarca et al. [22] outline the more recent developments. But despite the significant research in SPL, it is reported that there is a gap between practice and academic literature in this area. Not many companies apply the complex concepts that exist in the literature. Several authors have mentioned the use of basic inventory management techniques for service parts. Ashayeri et al. [23] found that their case, a company in the IT sector, used the EOQ (Economic Order Quantity) policy for consumable service parts inventory management, and it had proved to be reliable enough. The survey in [24] also indicates that basic, understandable inventory management techniques, including the EOQ model, are used commonly for service parts. Similarly, Huiskonen [25] reports that most basic inventory theory and models have been widely applied in practice and there is relatively little evidence of the use of more sophisticated applications, such as multi-echelon models.

The aim of this work is to provide insights into the relationship between service times, distribution setup, and inventory and transportation costs in a system where parts/materials are supplied within different service time windows. In scenarios where there are multiple levels of service, there can be two logical options to meet the requirements of all customers. One is to set up a system that has a uniform capability of meeting the toughest requirement for the entire customer base. The other is to set up a system that has variable capabilities to meet different requirements determined by different customers. Apparently, apart from being less complex, operating a system with a uniform capability (the first option) is unappealing. A uniform capability, which is tuned to meet the most stringent customer requirement, can result in overspending and does not include any mechanism to transfer cost benefits to customers requiring relaxed services. The problem is analysed by developing an estimate cost model considering a stylized system under two distinct organizations of stocking facilities, namely a hierarchical organization and a non-hierarchical organization.

Many real-world location problems involve facility systems that are hierarchical in nature, providing multiple levels of service through different types of facilities [26]. Expanding a location problem to consider more than one level of facility though increases the complexity of the problem [27, 28, 29]. Şahin and Süral [29] and, comparatively recently, Farahani et al. [30] have surveyed and classified the literature on hierarchical facility location problems. The objective of a hierarchical location model can be to locate different types of facilities and allocate them different types of customers such that some cost is minimized [e.g. 31], maximum number of customers are covered [e.g. 32], or all customers are covered [e.g. 33]. The last objective is rarely investigated, so is the consideration of continuous location. The hierarchical facility systems can themselves be classified in different ways; a major classification being nested and non-nested systems. In a nested hierarchy, a higher-level facility provides all the services provided by a lower-level facility and at least one additional service. While in a non-nested hierarchy, facilities at each level offer different services. Chistaller's hierarchical central places is a classical nested hierarchical system having a hexagonal pattern in a plane with continuous customer spread [34, 35]. In this system (Figure 1), 'central places of higher order' are defined as those that serve in a bigger region, in which other central places exist. In a higher level, not only the services of the higher order are offered, but those of the lower orders are also offered. The system comprises different circular ranges of central places depending on their types. To serve the entire land, a perfect and uniform net of central places is created, resulting in a hierarchical hexagonal pattern.

Figure 1: Circular range limits in central places system (from [36])



Similar to Chistaller's hierarchical central places, this paper considers a nest hierarchical organization of service areas to exploit the opportunity to centralize the distribution for demand for longer service time windows. This is compared with a non-hierarchical system, having the maximum level of decentralization and operating in a uniform fashion by distributing from the closest stocking facility regardless of the requested service time. The EOQ and the one-for-one (S-1, S) inventory policies are incorporated to investigate high and low demand rates respectively. The analysis focuses on the decentralization-centralization trade-off under multiple distance constraints and provides novel managerial insights into the service time and service cost relationship by considering the spatial aspect.

The remainder of the paper is organized as follows. Section 2 describes the problem and lists the assumptions made for the model formulation. Section 3 presents the model formulation under the hierarchical and the non-hierarchical setups. The formulation is followed by the analysis in Section 4 based on various numerical experiments. The paper is concluded in Section 5.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

Consider a set of demand points uniformly spread over a large geographical area. Customers have to be supplied with parts within different contracted service time commitments. A service type is associated with a particular maximum duration or a time window, translated into a *service distance constraint*, to deliver a part at a demand location. The strictness of a service type relates to how short the service distance constraint of the service type is; the *strictest* service being the one with the shortest service distance constraint. *Demand fractions* correspond to the proportion of total demand linked with different service types, such that the sum of all demand fractions is 1. In order to meet a service time commitment in the entire service area, every client location should be within the corresponding service distance constraint from at least one service parts storage facility. From now on, the paper refers to storage facilities, where parts are stored and from where they are dispatched to customers, as just *facilities*. Facilities are located to cover the entire area efficiently considering the strictest service's distance constraint. Under the *non-hierarchical setup* all facilities offer the full service range in their service areas. The facilities serve their vicinities, in effect without differentiating the required service time by a particular service request. Under the *hierarchical setup*, though all facilities provide the strictest service, only a subset of facilities provide services with longer distance constraints, in larger service areas, leading to a higher level of centralization in the system.

The aim is to determine the impact of changing the service distance constraints and demand fractions of different service types on the distribution network setup and inventory and transportation costs. Following assumptions are made to study the problem under the hierarchical and the non-hierarchical (completely decentralized) setups:

1. The EOQ model is considered for inventory ordering. It is a commonly used model for inventories and allows a tractable formulation to convey the main insights. A cross industry exploratory investigation in [37] shows that many of the predictions from classical inventory models, such as the EOQ model, extend beyond individual products to the aggregate firm level. Hence, these models can help with high-level strategic choices in addition to tactical decisions.
2. The problem is also formulated and analysed under the (S-1, S) inventory policy, which is considered to be appropriate for slow moving items and is a widely noted inventory policy in the SPL literature. The following commonly used assumptions are considered for the (S-1, S) policy: single item; demand arrives one at a time according to the Poisson process; backorders allowed; and no capacity constraints on the supply (replenishment). It is pertinent to mention that under the EOQ based (R, Q) policy, with a constraint on stock-out probability during the lead time, the order quantity Q can be determined as the EOQ and the reorder point R can be determined in the same way as the base stock S is computed under the (S-1, S) policy.
3. Demand is assumed to have a uniform spread over the area, which is to be covered by a single-echelon distribution system. It can be argued that considering a demand area rather than a finite set of demand points is more realistic as demand may move and increase or decrease. It is also assumed that travelling distances are Euclidean and proportional to travel time. Examples of studies with such simplifying assumptions can be found in [36].
4. It is assumed that the area to be covered to provide services is large and that the services have to be provided within short time commitments. Typically, large IT companies cover vast geographical areas for service parts distribution through several service facilities. Considering that the facility costs are dominating, the system comprises the minimum number of facilities that can provide the complete demand coverage considering a service time constraint. Cohen et al. [16] suggest that having a high number of storage facilities, when the customer need is not urgent, is a mismatched SPL strategy. Responsiveness comes at a significant cost in a typical SPL system [12].
5. A regular hexagonal packing of service catchment areas is considered, assuming that facility points are located efficiently to cover the entire area for the strictest service time commitment, and that a service request is fulfilled by the nearest service facility offering the required service. The hexagonal partitioning is considered to be efficient when assuming Euclidean distances. Out of the shapes that can tile a plane with perfect packing, hexagons cover the maximum area considering a distance constraint from the centre, and have the minimum average distance to the centre considering a fixed area size [38]. Okabe et al. [36] include a Voronoi diagram based computational method for determining the efficient location and service areas of given number of non-hierarchical facility points in an area with different demand density functions. Their approximate optimum solutions are hexagonal patterns of service areas with different concentrations of facilities depending on the density functions. Examples of service system studies that consider a regular hexagonal pattern of service catchment areas for their analysis include [39, 40].
6. For an analytical treatment, boundary effects and rounding off errors are ignored in determining the number of facilities. Hence the analysis is an approximation. To calculate the number of facilities the total area is divided by a facility catchment area (a full hexagonal area determined according to the maximum distance that can be travelled within a committed service time), which can result in a fractional number. Secondly, packing an area in a plane with full identical hexagons may not provide complete coverage, and typically, areas on the boundaries of the region may need to be covered by partial hexagons and need additional facilities. Simply dividing an entire service area with a facility catchment area can underestimate the number of facilities required for the coverage. Nevertheless, the boundary effect becomes less significant in case of a large overall area covered by a high number of regular hexagons [38, 41]. It can be shown that the boundary effect diminishes when the overall area becomes large, and hence, the estimations from the cost formulae improve.

3. THE MODEL

The model represents non-hierarchical and hierarchical setups of facilities providing multiple time-based service types, where a *stricter* service has a shorter time window for a delivery. Let Type *1* service be the least strict service and Type *k* service be a stricter service than all Type $k - i$ services, where $2 \le k \le m$ *and* $1 \le i < k$. Hence *m* is the total number of service types and a Type *m* service is the strictest service.

Below is the list of the notations used for formulating the demand compositions under the non-hierarchical and the hierarchical setups:

A: total area to be served (a large geographical area)

λ: total demand in area *A*, i.e. the total number of service calls per unit time

m: total number of service types

f_k: fraction of total demand corresponding to Type *k* service, where $\sum_{k=1}^{m} f_k = 1$

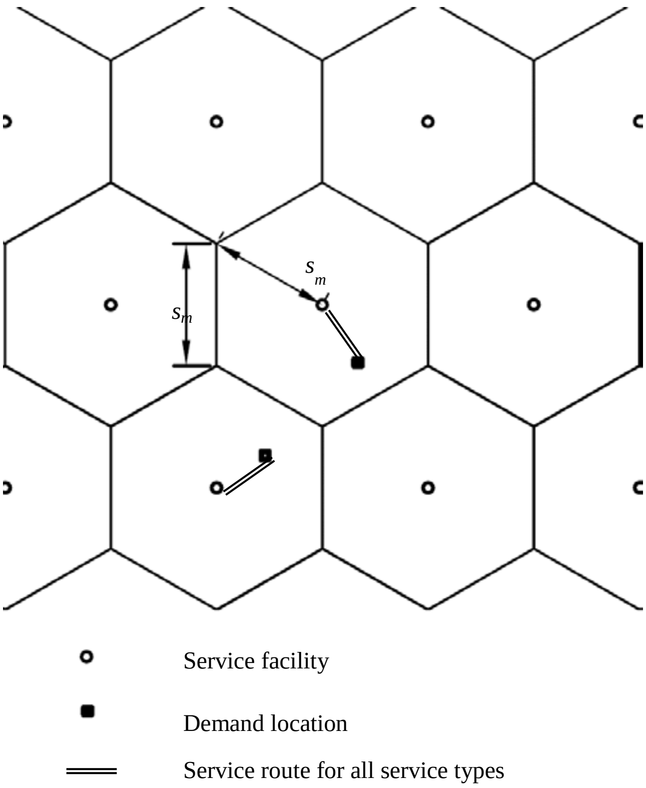
s_k: service distance constraint for Type *k* service, where $k = 1 \ldots m$ and s_m is the shortest distance constraint

n_k: number of facilities providing Type *k* service, where $k = 1 \ldots m$ and n_m is the total number of facilities in the system

3.1 Non-hierarchical setup of service facilities

The non-hierarchical setup consists of only one facility type. All service facilities provide the full range of service types, i.e. all Type *k* services, where $k = 1 \ldots m$. All facility catchment areas are marked considering the maximum distance that can be covered from a service facility to provide the strictest (Type *m*) service (Figure 2).

Figure 2: Non-hierarchical setup



As sm is the maximum distance that can be covered from a service facility to provide a Type m service, it is also equal to the edge length of the hexagonal service catchment areas of a facility (Figure 2). The total number of facilities (nm) is determined as $\frac{A}{3\sqrt{3}(s_m)^2/2}$, where $3\sqrt{3}(s_m)^2/2$ is the hexagonal catchment area of a service facility with an edge length of sm. As each facility provides the complete set of service types, nk is the same for k = 1 … m. Consequently, the total demand served by one facility is λ/nm.

3.2. Hierarchical setup of service facilities

A nested hierarchy of facilities and service areas is considered to present a system that avails itself of the opportunity to meet the demand for relaxed services in a centralized fashion. A nested hierarchical hexagonal pattern can be generated by locating the centres (facility points) of lower level hexagons 1) at the middle of the edges of the higher level hexagons, or 2) at the corner points of the higher level hexagons (Figure 3). These two approaches result in different ratios between the maximum distances (or service time constraints) within the higher and lower level hexagons.

Figure 3: Hierarchical hexagonal pattern

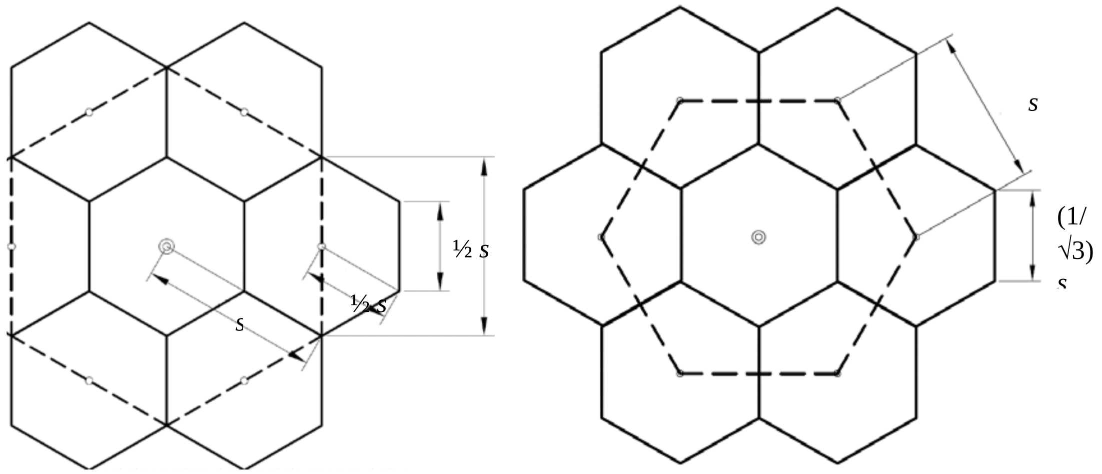


Figure 4 presents a combination of the two approaches of locating lower level hexagons with respect to higher level hexagons.

Figure 4: Mixed hierarchical hexagonal pattern

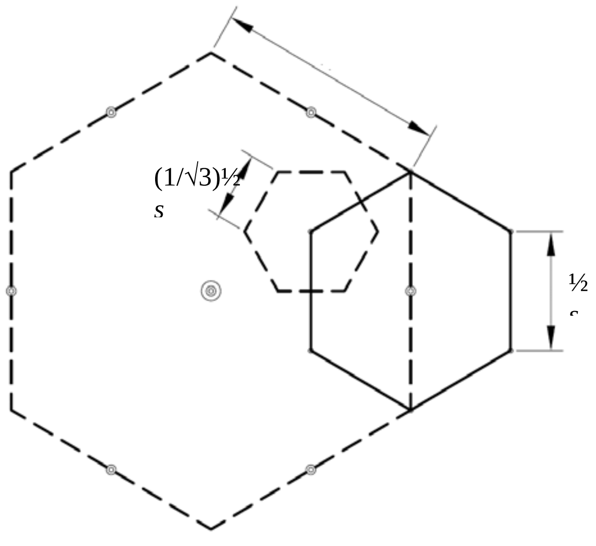
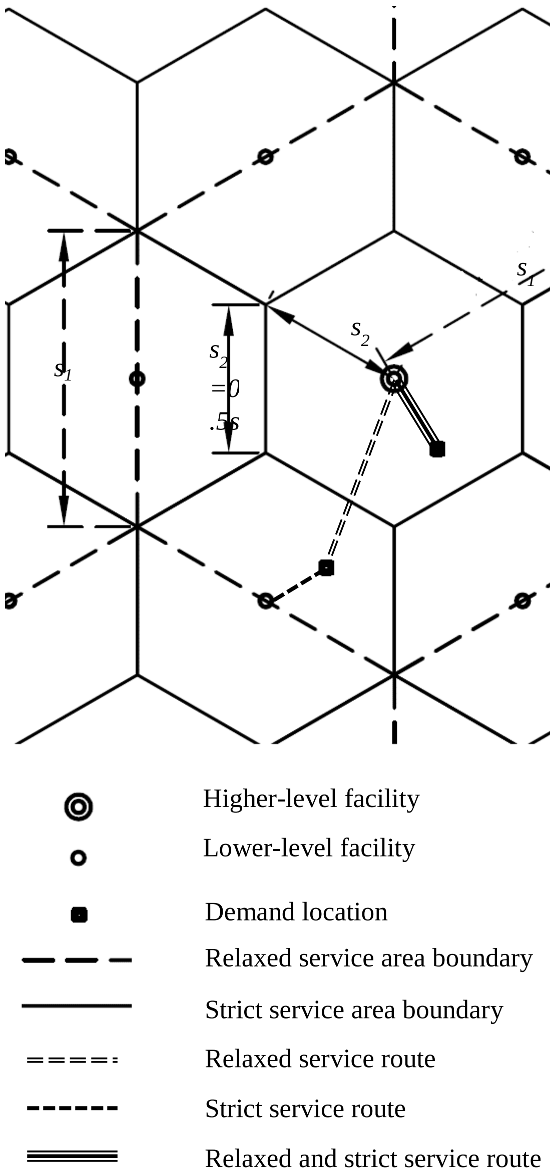


Figure 5 depicts the hierarchical system considering two service types; the strict service and the relaxed service (the latter having the longer distance constraint). Though the figure depicts a hierarchical hexagonal pattern in which the centres of lower level hexagons are located at the middle of the edges of higher level hexagons, the formulation can also be used to study the service time ratios resulting from the other pattern. The ratio between the different service time constraints that the pattern in Figure 5 allows is in line with the ones offered by the real-world IT SPL systems that the author has studied.

Figure 5: Two level Hierarchical setup



The hierarchical setup providing two service time options consists of two types of service facilities; the *higher-level* facilities providing both the relaxed and the strict services, and the *lower-level* facilities only provide the strict service. A higher-level facility has two service catchment areas; the first within the strict service distance constraint and the second within the relaxed service distance constraint. All customers within the first catchment area of a higher-level facility can get both types of service from the higher-level facility, whereas the customers beyond the first catchment area and within the second catchment area can only get the relaxed service. Unlike higher-level facilities, a lower-level facility has only one catchment area, which is within the strict service distance constraint. A lower-level facility can only provide the strict service to the customers within its catchment area. For the relaxed service, the customers within a lower-level facility's catchment area are served by the closest higher-level facility.

From the notations, s_1 and s_2 are the maximum distances that can be covered from a service facility to provide the relaxed and strict services respectively. Consequently, s_1 and s_2 are equal to the edge length of a hexagonal catchment area for providing the relaxed and the strict services respectively. In the hierarchical setup presented in Figure 5, the maximum distance that can be travelled in a straight line from a lower-level facility is half of the maximum distance that can be travelled from a higher-level facility for the relaxed service. With such placement, assuming Euclidean distances, if the time constraint for the relaxed service is 8 hours, the possible time constraints for the strict service can be 4 hours, 2 hours, 1 hour and so on.

A generic formulation is now presented for a nested hierarchical system providing two or more time-based service types. The number of facilities providing Type *k* service, i.e. n_k, is determined as $\frac{A}{3\sqrt{3}(s_k^{\,2})/2}$, where $3\sqrt{3}(s_k^{\,2})/2$ is the hexagonal catchment area of a service facility for Type *k* service provision. A higher-level facility provides a more relaxed service in addition to all the services provided by the lower-level facilities. Let Type *1* facilities be the highest level facilities providing the complete set of services. Let Type *k* facilities be lower-level facilities than all Type $k - i$ service facilities, where $2 \leq k \leq m$ and $1 \leq i < k$, providing a Type *k* service and all the services that are stricter than Type *k* service (Table 1). Hence a Type *m* facility is the lowest level facility, only providing Type *m* (the strictest) service, and *m* is the total number of service facility types. For instance, assuming a successively inclusive hierarchical system providing three service types, a Type *1* service is the least strict service, a Type *2* service is stricter than Type *1* service, and a Type *3* service is the strictest service. In terms of the facility hierarchy, Type *1*, Type *2*, and Type *3* are the types of facilities in descending order of their level – Type *1* being the highest level facilities and Type *3* being the lowest.

Table 1: Classification of service facilities in nested-hierarchical system

Type *1* service facilities provide →	✓	✓	✓	…	✓
Type *2* service facilities provide →		✓	✓	…	✓
Type *3* service facilities provide →			✓	…	✓
⋮					⋮
Type *m* service facilities provide →					✓

Considering the classification of service facilities presented above, the number of Type *k* facilities is estimated as $n_k - n_{k-1}$, where $k = 1 \ldots m$ and $n_0 = 0$. For example, the number of Type *3* facilities (i.e. k = 3) equals $n_3 - n_2$, where n_3 is the number of facilities providing the Type *3* service, and n_2, the sum of the numbers of Type *1* and Type *2* facilities, is the number of facilities providing a Type *2* service.

Table 2 shows the expressions for the total demand served by the facilities of each type.

Table 2: Demand composition in the nested-hierarchical system

Total demand served by Type *1* service facilities =	$f_1\lambda$	$+\frac{n_1}{n_2}f_2\lambda$	$+\frac{n_1}{n_3}f_3\lambda$	…	$+\frac{n_1}{n_m}f_m\lambda$
Total demand served by Type *2* service facilities =		$\frac{n_2-n_1}{n_2}f_2\lambda$	$+\frac{n_2-n_1}{n_3}f_3\lambda$	…	$+\frac{n_2-n_1}{n_m}f_m\lambda$
Total demand served by Type *3* service facilities =			$\frac{n_3-n_2}{n_3}f_3\lambda$	…	$+\frac{n_3-n_2}{n_m}f_m\lambda$
⋮					⋮
Total demand served by Type *m* service facilities =					$\frac{n_m-n_{m-1}}{n_m}f_m\lambda$
Total demand =	$f_1\lambda$	$f_2\lambda$	$f_3\lambda$		$f_m\lambda$

From Table 2, total demand served by all Type k service facilities is

$$\sum_{i=k}^{m}\frac{n_k-n_{k-1}}{n_i}f_i\lambda \text{ where } n_k-n_{k-1}$$

is the total number of Type k facilities such that n_0 is 0. Consequently, total demand served by one Type k facility is

$$\frac{1}{n_k-n_{k-1}}\sum_{i=k}^{m}\frac{n_k-n_{k-1}}{n_i}f_i\lambda=\sum_{i=k}^{m}\frac{f_i}{n_i}\lambda . \quad (1)$$

3.3 Cost formulations

The cost functions are formulated considering the expressions for the demand composition presented above (Sections 3.1 and 3.2), the EOQ policy, and the average distance to reach customers in a hexagonal area. The problem is studied considering the one-for-one (S-1, S) inventory policy in Section 3.4. The following additional notations are introduced:

- λ: total demand in area A, i.e. the total number of service calls per unit time
- r: cost per inventory replenishment order
- h: holding cost per unit per unit time
- t: transportation cost per unit distance
- T_{ICNH}: total inventory cost per unit time under the non-hierarchical setup
- T_{ICH}: total inventory cost per unit time under the hierarchical setup
- T_{TCNH}: total transportation cost per unit time under the non-hierarchical setup
- T_{TCH}: total transportation cost per unit time under the hierarchical setup

Considering the EOQ policy in a centralized system (having only one facility), the total inventory order and holding cost per unit time is $\sqrt{2rh\lambda}$. Within a decentralized system, if λ_i is the demand served by service facility i, where $i = 1 \dots n$, then assuming that each facility applies the EOQ policy, the total cyclic inventory cost per unit time is $\sum_{i=1}^{n}\sqrt{2rh\lambda_i}$.

Under the non-hierarchical setup, since λ/n_m is the demand served by one facility (Section 3.1), $\sqrt{2rh\frac{\lambda}{n_m}}$ is the inventory cost per unit time at one facility. With n_m as the total number of facilities we get the following inventory cost estimation function for the non-hierarchical setup:

$$T_{ICNH}=n_m\sqrt{2rh\frac{\lambda}{n_m}}=\sqrt{2rh\lambda n_m} \quad (2)$$

Under the hierarchical setup, considering the demand served by one Type k facility (1), and the number of Type k facilities $n_k - n_{k-1}$, the total inventory cost per unit time at Type k facilities is estimated as $(n_k-n_{k-1})\sqrt{2rh\sum_{i=k}^{m}\frac{f_i}{n_i}\lambda}$. Summing the inventory cost at all m types of facilities, we obtain:

$$T_{ICH}=\sum_{k=1}^{m}\left((n_k-n_{k-1})\sqrt{2rh\sum_{i=k}^{m}\frac{f_i}{n_i}\lambda}\right)$$

For a two-level hierarchy, i.e. $m = 2$, the function becomes:

$$T_{ICH} = n_1\sqrt{2rh\left(\frac{f_1}{n_1}+\frac{f_2}{n_2}\right)\lambda} + (n_2-n_1)\sqrt{2rh\frac{f_2}{n_2}\lambda}$$

$$T_{ICH} = \sqrt{2rh\lambda n_1}\left(\sqrt{f_1+\frac{n_1}{n_2}f_2}+\left(\frac{n_2}{n_1}-1\right)\sqrt{\frac{n_1}{n_2}f_2}\right) \tag{3}$$

Multiplication factor (T_{ICH})*:*

$$\sqrt{f_1+\frac{n_1}{n_2}f_2}+\left(\frac{n_2}{n_1}-1\right)\sqrt{\frac{n_1}{n_2}f_2} \tag{4}$$

Considering the two level hierarchical system, providing a strict and a relaxed service, three extreme cases, or benchmarks, of the inventory cost can be identified. 1) When there is only one facility providing the service (i.e. a completely centralized system), the cyclic inventory cost, as mentioned earlier, equals $\sqrt{2rh\lambda}$. 2) When there is no demand for the strict service, i.e. $f_2 = 0$ and $f_1 = 1$, the multiplication factor (4) equals 1, reducing the total inventory cost function (3) to $\sqrt{2rh\lambda n_1}$, i.e. the cost increases by the factor of $\sqrt{n_1}$ compared to the completely centralized case. 3) When there is no demand for the relaxed service, i.e. $f_1 = 0$ and $f_2 = 1$, and hence the system becomes completely decentralized, the multiplication factor (4) becomes $\sqrt{n_2/n_1}$, reducing the inventory cost function to $\sqrt{2rh\lambda n_2}$. In other words, the cost increases by the factor of $\sqrt{n_2}$ compared to the completely centralized case.

When considering Euclidean travelling distances, a uniform geographical distribution of customers, and hexagonal service areas with facilities located at their centres, the average distance to reach a customer is $(1/3 + \ln 3/4)s \approx 0.60799(s)$, where s is the edge length of the hexagonal service area (see [42] for details on the average distance). Under the non-hierarchical setup, since s_m is the edge length of a service area, the average distance to reach a customer is estimated as $0.60799(s_m)$. Subsequently, the following function estimates the transportation cost under the non-hierarchical setup:

$$T_{TCNH} = 0.60799\ s_m\lambda t \tag{5}$$

Under the hierarchical setup, considering $0.60799(s_k)$ as the average distance to serve a customer for a Type k service, where s_k is the edge length of a hexagonal service area for Type k service, the following function estimates transportation cost:

$$T_{TCH} = 0.60799\sum_{k=1}^{m} f_k\lambda s_k t$$

The transportation cost function and the multiplication factor for the two-level system are:

$$T_{TCH} = 0.60799(f_1\lambda s_1 + f_2\lambda s_2)t = 0.60799\lambda s_1 t\left(f_1 + f_2\frac{s_2}{s_1}\right) \tag{6}$$

Multiplication factor (T_{TCH})*:*

$$\left(f_1 + f_2\frac{s_2}{s_1}\right) \tag{7}$$

In the two level system, when there is no demand for the strict service ($f_2 = 0$) the multiplication factor for the transportation cost (7) equals to 1 and the transportation cost becomes independent of the ratio between the strict and the relaxed service times. With the presence of demand for the strict service ($f_2 > 0$), the smaller the distance constraint for the strict service, resulting in smaller strict service areas, the lower the multiplication factor will be.

Total operating cost per unit time under the non-hierarchical setup is:

$$T_{ICNH} + T_{TCNH} \tag{8}$$

While, under the hierarchical setup the cost is:

$$T_{ICH} + T_{TCH} \tag{9}$$

3.4 Considering the (S-1, S) inventory policy

Finally, let L be the replenishment lead time and S be the base-stock level at a facility with the (S-1, S) inventory policy. The value of S can be determined through the steady state probability of the quantity of units in resupply. Note that there is a well-known relationship between stock-out probability and fill-rate (fraction of demand met from stocks on hand) under the (S-1, S) inventory policy [20, 43, 44]. Considering that all the demand is met by one facility, the fill-rate under the (S-1, S) policy is equal to the probability that demand is less than S over the replenishment lead time ($P(\lambda L<S)$), which is $\sum_{x<S}\frac{e^{-\lambda L}(\lambda L)^x}{x!}$, where $\frac{e^{-\lambda L}(\lambda L)^x}{x!}$ is the unconditional probability that x units remain in the resupply. The value of S is set as the minimum integer value for which $P(\lambda L<S)$ is greater than or equal to the required fill-rate.

For the non-hierarchical setup, the base stock level at one facility is determined considering the demand over lead time faced by one facility ($L\lambda/n_m$) and a minimum required fill-rate level. The total base stock level is then estimated by multiplying the base stock level at one facility by the total number of facilities:

$$n_m S \quad (10)$$

Under the hierarchical setup, the total base stock level at Type k facilities is estimated as the product of the number of Type k facilities $n_k - n_{k-1}$ and the base stock level S_k at one Type k facility. The base stock level S_k is computed considering $L\sum_{i=k}^{m}\frac{f_i}{n_i}\lambda$ as the demand over lead time at one Type k facility, where $\sum_{i=k}^{m}\frac{f_i}{n_i}\lambda$ is the demand faced by one Type k facility over the unit time (1). The total base stock level in the system is then estimated as the sum of the base stock levels at all facility types:

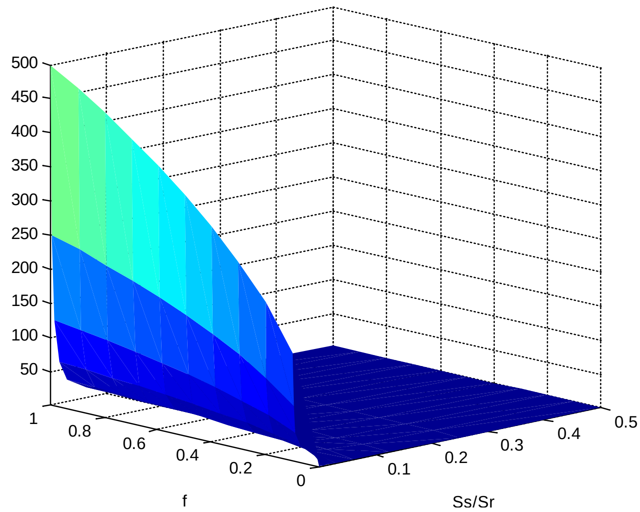
$$\sum_{k=1}^{m}(n_k - n_{k-1})S_k \quad (11)$$

4. NUMERICAL ANALYSIS

The analysis considers two service types, referred to as the *strict* and the *relaxed* services. Type *1* and Type *2* facilities in the hierarchical setup are referred to as *higher* and *lower*-level facilities respectively. The multiplication factors for inventory and transportation costs under the hierarchical setup are analysed first and then the non-hierarchical and the hierarchical setups are investigated and compared over different service distance constraints and demand compositions.

Figure 6, based on the multiplication factor for the inventory cost (4), and Figure 7, based on the multiplication factor for the transportation cost (7), illustrate how inventory and transportation costs react to the changes in s_2/s_1 ratio (0.5, 0.25, … 0.002) and the demand fractions for the relaxed and strict services. A smaller s_2/s_1 represents a greater time difference between both service types, requiring comparatively more lower-level facilities and a higher level of decentralization. Likewise, a higher value of f_2 (the proportion of demand for the strict service) means that more demand has to be fulfilled from lower-level facilities, which are more decentralized. Note that the non-hierarchical setup does not provide an opportunity to perform this type of analysis as entire demand, whether for the strict service or the relaxed service, is met in a similar way (from the same facilities). This is discussed further in the examples.

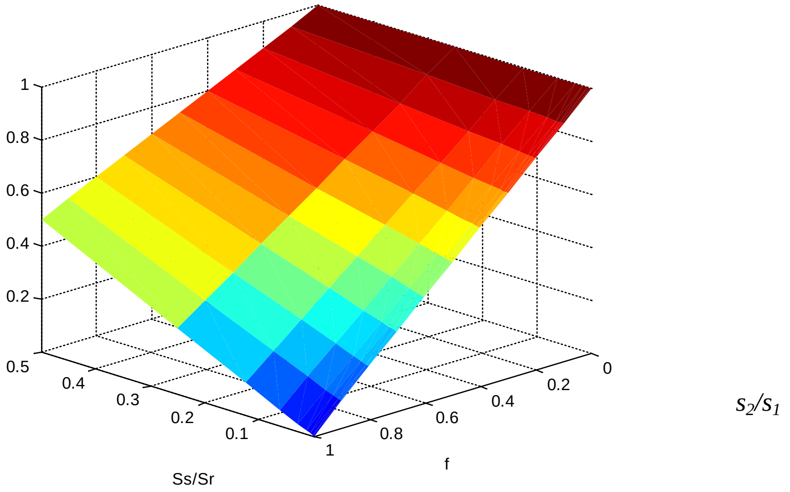
Figure 6: Multiplication factor for inventory cost



f_2 s_2/s_1

Inventory in a system increases when s_2/s_1 decreases and/or f_2 increases (Figure 6). The maximum inventory is maintained when s_2/s_1 is at its minimum and f_2 is maximum (i.e. $f_2 = 1$). With s_1 constant, a smaller s_2/s_1 ratio means that more lower-level service facilities have to be set up to satisfy the service distance constraint for the strict service. With maximum f_2 there is no demand for the relaxed service. This requires stocks to be maintained with the maximum decentralization as there is no allowance to meet demand from longer distances. In this scenario, a system under the hierarchical setup operates similarly to when it is under the non-hierarchical setup. When f_2 is at its minimum (i.e. $f_2 = 0$), the inventory levels are constant and minimum as the total demand, composed of only relaxed service calls, is met within the larger service areas of more centralized higher-level facilities.

Figure 7: Multiplication factor for transportation cost



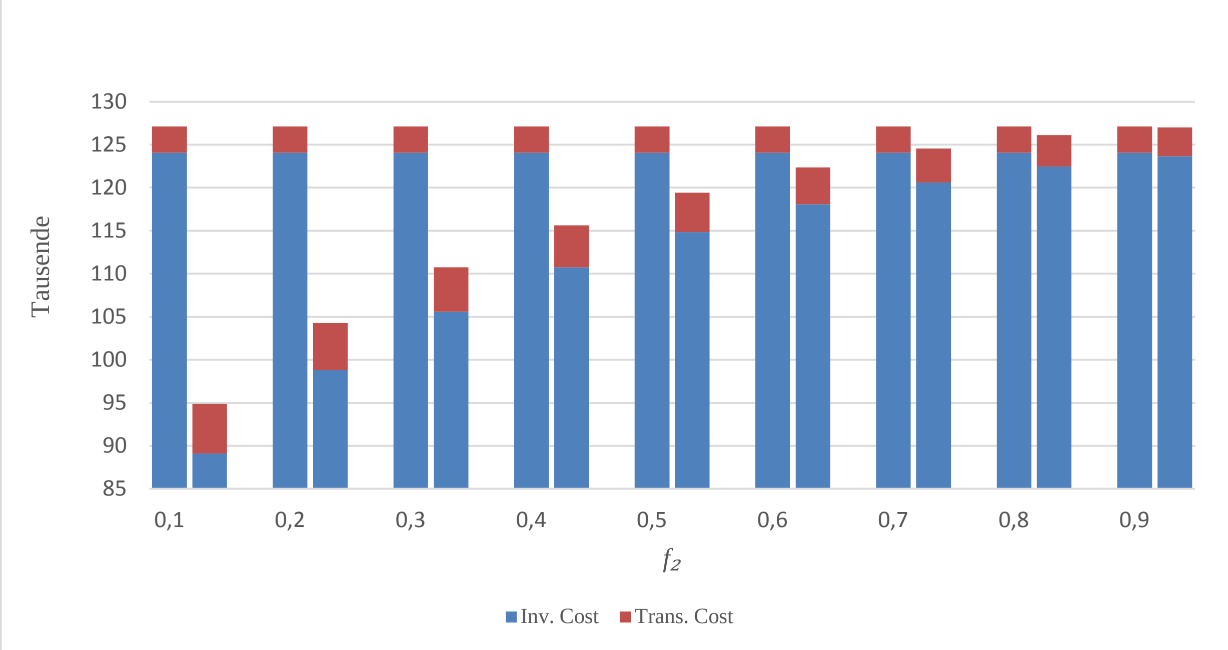
In contrast to inventory levels, travelling reduces when s_2/s_1 decreases (Figure 7). The minimum transportation cost results when s_2/s_1 is at its minimum and f_2 is at its maximum (i.e. $f_2 = 1$). As stated earlier, a smaller s_2/s_1 ratio results in more lower-level facilities and maximum f_2 results in the total demand being fulfilled by facilities in smaller catchment areas, which in turn results in lower average travelling to serve customers. Transportation cost is at the maximum level when $f_2 = 0$, irrespective of the s_2/s_1 value, because regardless of the number of lower-level facilities, the total demand is served by higher-level facilities within larger service areas.

The results from the computations based on the following hypothetical values are presented to further illustrate and compare the hierarchical and non-hierarchical setups. Let demand (λ) = 10,000 per year, area (A) = 200,000-unit length^2 (the area of the mainland UK being 229,543 km^2), and the relaxed service distance constraint (s_1) = 100-unit length. Two sets of cost parameters values are considered to represent a case where inventory cost

dominates and a case where transportation cost dominates. In **Case 1**, with dominating inventory cost, r = 500, h = 50, and t = 0.01. In **Case 2**, r = 100, h = 10, and t = 0.2. The values for the strict service distance constraint (s_2) and the demand fractions for both service types (f_2 *and* f_1) are altered for the analysis.

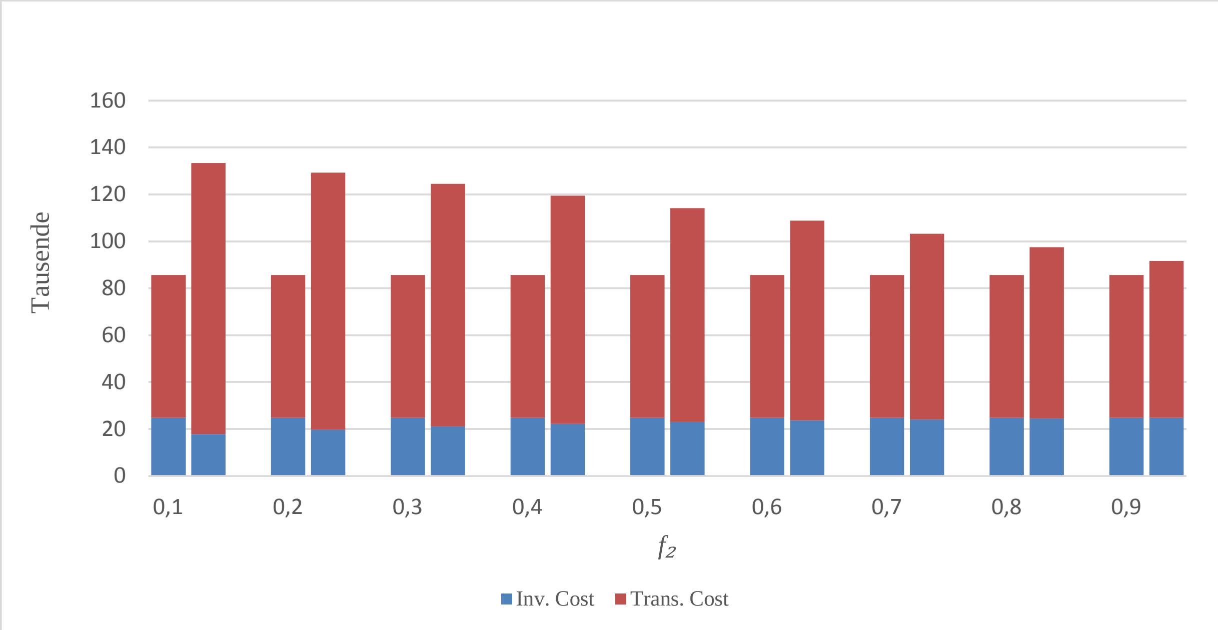
Figure 8 presents the total cost calculations (equations (8) and (9)) for varying f_2, while retaining s_1 = 100 and s_2 = 50-unit lengths. Under the non-hierarchical setup, the inventory cost (2) and the transportation cost (5) remain unaffected by the changes in the proportions of demand for both service types. Under the hierarchical setup, a lower proportion of demand for the strict service results in a lower inventory cost (3) while a higher transportation cost (6). The hierarchical setup performs better when the inventory costs dominate (Figure 8a) as opposed to when the transportation cost dominates (Figure 8b). It can exploit the opportunity to respond to relaxed service calls with a higher level of centralization, as a longer distance can be covered for a delivery. Under the non-hierarchical setup, on the other hand, the system cannot exploit this opportunity and supplies items to customers with a constant level of decentralization, i.e. from the closest facility. Higher fraction of demand for the relaxed service results in more centralized demand fulfilment within larger service areas under the hierarchical setup, which reduces the inventory but increases the average service distance. The costs under both setups converge when the fraction of demand for the strict service increases; both setups operating in the maximum decentralized fashion when f_2 is 1.

Figure 8: Impact of demand fractions for the strict and relaxed services on costs ($s_1 = 100$; $s_2 = 50$)



NH: Non-hierarchical; H: Hierarchical

a) Case – 1: Dominating inventory costs



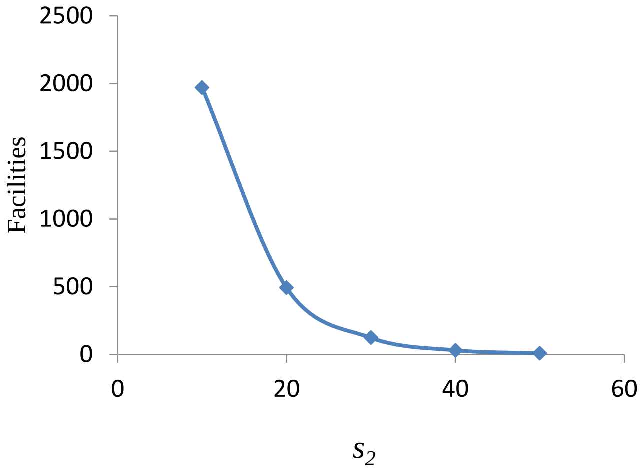
NH: Non-hierarchical; H: Hierarchical

b) Case – 2: Dominating transportation cost

There are two inventory and transportations cost benchmarks under the hierarchical setup. The first is at the point where there is no demand for the strict service ($f_2 = 0$), hence the service is only provided from the higher-level facilities in their larger service areas, resulting in the lowest inventory cost and the highest transportation cost in the system. The second is at the point where there is no demand for the relaxed service and hence the system operates as a non-hierarchical system, resulting in the highest inventory cost and the lowest transportation cost. Both costs vary between these two levels depending on the value of f_2.

Figs. 9 and 10 show the impact of service distance constraint on the distribution system and the service cost. The reduction in the service time constraints when s_2 is small sharply increases the number of required facilities and hence the setup cost (Figure 9).

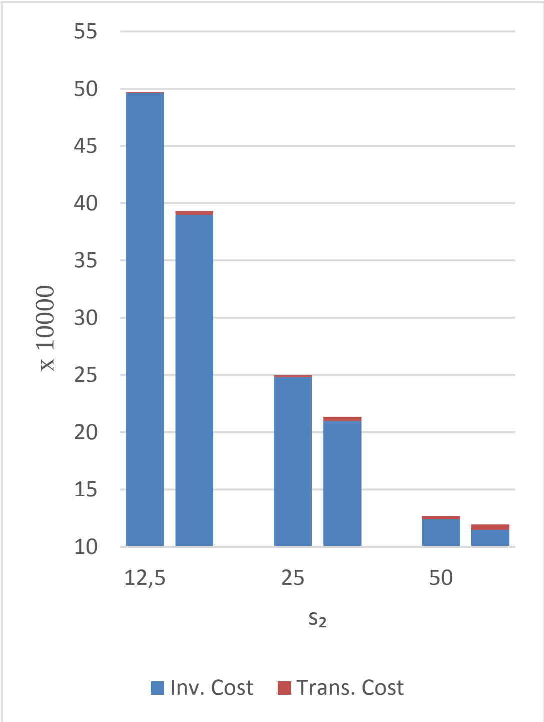
Figure 9: Impact of service distance constraint on the number of facilities



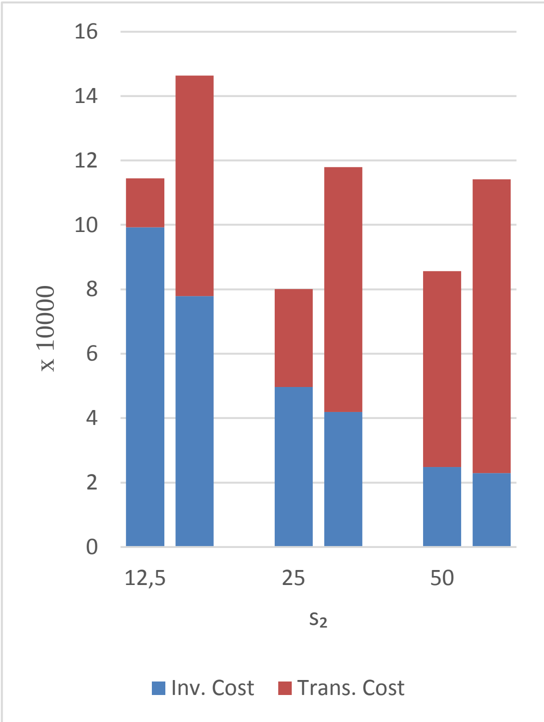
When the distance constraint for the strict service decreases (keeping constant the relaxed service distance constraint s_1 and the proportions of demand for both service types, f_1 and f_2), the inventory cost under both setups increases, and so does the difference by which the hierarchical setup performs better than the non-hierarchical setup in terms of the inventory cost (Figure 10). In contrast to the inventory cost, the amount by which the non-hierarchical setup performs better than the hierarchical setup in terms of the transportation cost increases as the strict service distance (s_2) decreases.

With a reduction in the time window for the strict service, the non-hierarchical setup meets both types of demand with a higher decentralization level, whereas under the hierarchical setup, the decentralization level for the relaxed service does not change, making the inventory and transportation costs under the hierarchical setup less sensitive to the changes in s_2. Overall, the hierarchical setup performs better in Case 1, in which inventory costs dominate, and the non-hierarchical setup performs better in Case 2, having a higher transportation cost.

Figure 10 Impact of service distance constraint on inventory and transportation costs (f_2 = 0.5; s_1 = 100)



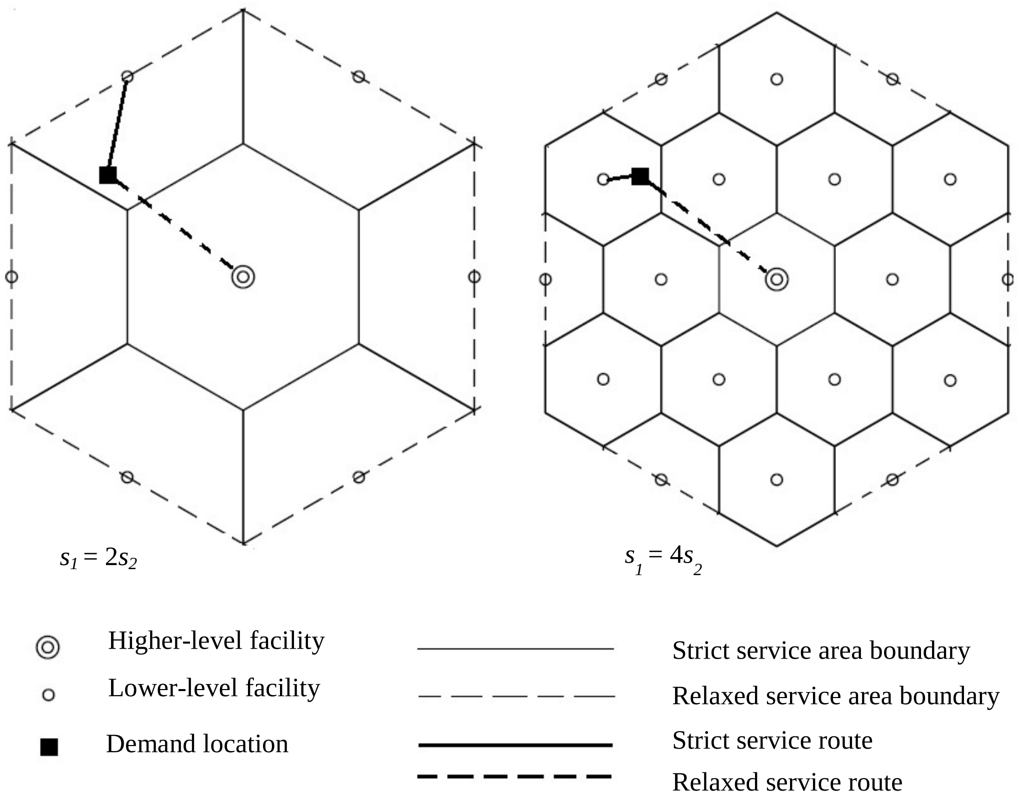
NH: Non-hierarchical; H: Hierarchical
a) Case – 1: Dominating inventory costs



NH: Non-hierarchical; H: Hierarchical
b) Case – 2: dominating transportation costs

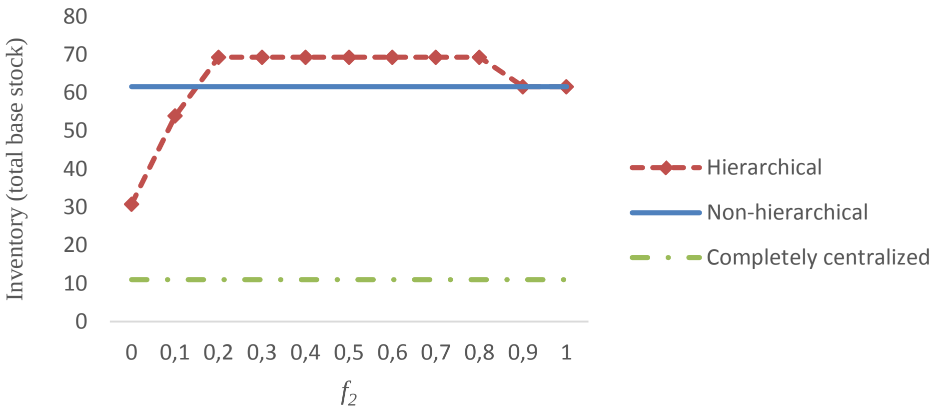
With higher fraction of demand for the strict service, more demand is fulfilled in the smaller strict service areas, and with the reduction in the strict service distance constraint, these smaller areas further reduce in size (Figure 11). Both these factors increase the level of decentralization, which adversely impacts the inventory cost performance while reducing the transportation cost.

Figure 11: Impact of service distance constraint on location pattern



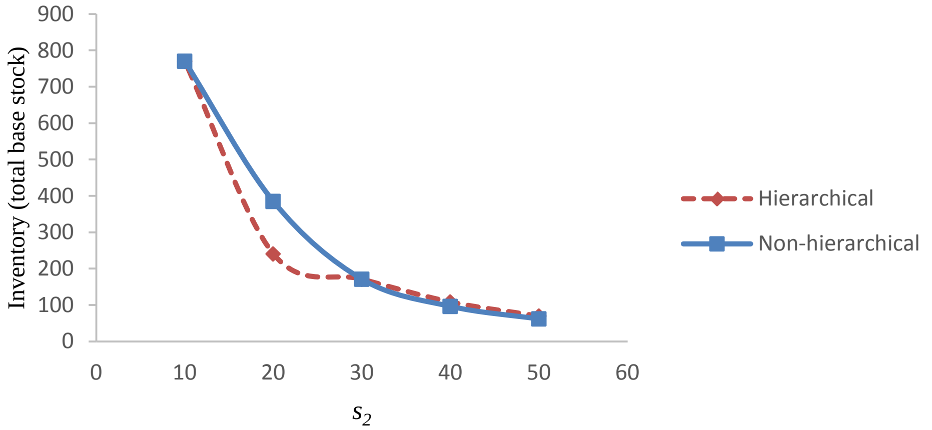
The final set of results is based on overall inventory level computations considering the (S-1, S) inventory policy under the modelled non-hierarchical and hierarchical setups. The computations consider the demand rate of 5 parts per day, fixing the replenishment lead-time (L) at 1 day, and considering the overall area as 200,000-unit length2. (Computations based on the demand rate of 1 part per day provided similar insights.) Figure 12 shows the effect of varying demand fractions for both services on inventory levels with the distance constraints for the relaxed and strict services as 100 and 50-unit lengths respectively, and the minimum fill-rate level as 0.98. The results suggest that inventory levels under the hierarchical setup (11) can be higher than under the non-hierarchical setup (10). Under the non-hierarchical setup, the demand over the lead time at a facility is 0.16, requiring the base-stock level to be 2 to satisfy the minimum fill-rate level of 0.98. As total number of facilities (n2) is 30.79, the total inventory under the non-hierarchical setup is considered as $\lceil 2\times30.79 \rceil = 62$. Under the hierarchical setup, when f2 = 0.1, the demand at a lower-level facility is 0.02, requiring the base-stock level to be 1, and the demand at a higher-level facility is 0.6, requiring the base-stock level to be 4. With n2 – n1 = 23.09 and n1 = 7.698, the total inventory is taken as $\lceil (1\times23.09) + (4\times7.698) \rceil = 54$. When f2 = 0.2, lesser demand is consolidated at the higher-level facilities (0.55 over the lead time), requiring 3 units as the base-stock, while more demand is shifted to the lower-level facilities (0.03 per unit time), increasing the required base-stock level to 2. This increases the overall inventory to 70, which is higher than the total inventory under the non-hierarchical setup. Increasing f2 to 0.9 increases the demand at a lower-level facility to 0.15, while decreasing the demand at a higher-level facility to 0.21. Both types of facilities consequently require base-stock levels of 2, resulting in the overall inventory of 62.

Figure 12: Impact of demand fraction for the strict and relaxed services on inventory (s_1 = 100; s_2 = 50)



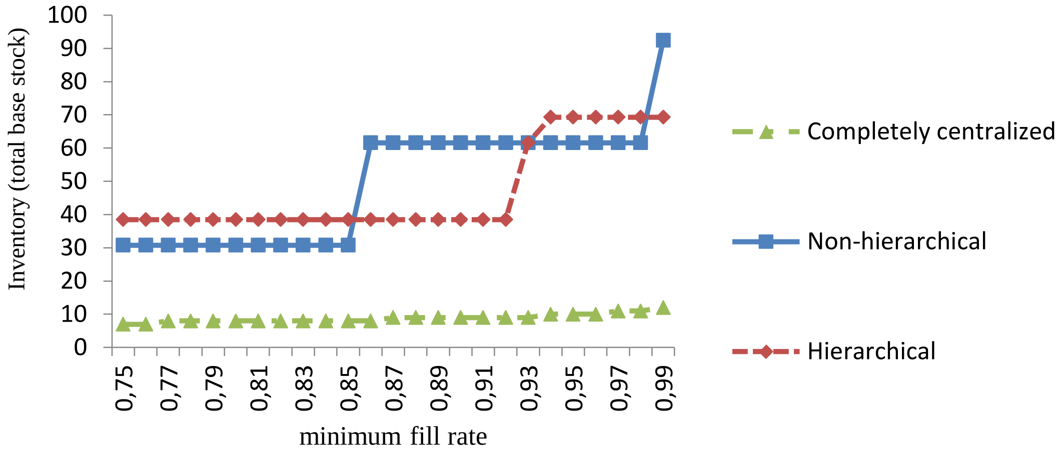
With equal fractions of demand for both service types and the minimum fill-rate level as 0.98, Figure 13 shows the impact of reducing service distance constraints (such that s_1 = 2s_2) on inventory levels. Both setups see an increase in their inventory levels as the distance constraint for the strict service reduces, resulting in a higher number of facilities and more decentralization. Again there are instances (when s_2 = 20 & 50) where the hierarchical setup results in more inventory compared to that under the non-hierarchical setup.

Figure 13: Impact of service time constraint on inventory (f_2 = 0.5; s_1 = 2s_2)



Lastly, Figure 14 presents the inventory level computations against varying fill-rate constraint. The inventory levels under the hierarchical and non-hierarchical setups increase stepwise when the minimum required fill-rate increases, with non-hierarchical setup performing better in several cases. The inventory level in a completely centralized system, where there is only one service facility, has very low sensitivity to the fill-rate constraint.

Figure 14: Impact of required minimum fill-rate level on inventory (s_2 = 50, s_1 = 100 and f_2 = 0.5)



The computations considering (S-1, S) policy provide an interesting observation. The results suggest that even though the level of centralization is higher under the hierarchical setup, the inventory levels under the completely decentralized non-hierarchical setup can be lower. The reason behind this phenomenon is that a slight reduction in demand at a facility does not always allow the facility's base stock level to be reduced. Although transforming a system from the non-hierarchical setup to the hierarchical setup reduces demand at the lower-level facilities, the required base stock levels at these facilities cannot necessarily be reduced while maintaining the minimum fill-rate level. On the other hand, the transformation increases demand at higher-level facilities and can potentially increase the required base stock levels at these facilities to maintain the minimum fill-rate level. Hence, on the whole, this can increase the stocks in the system. The SPL systems of two major multinational IT equipment manufacturers that the author has studied operate in a non-hierarchical way. The above analysis shows that a non-hierarchical setup, besides being simpler, is not necessarily inferior to a hierarchical setup.

5. SUMMARY AND CONCLUSION

The presented model and analysis explore the effects of different service time constraints on some of the important components of distribution cost. The results presented in the preceding sections show how service time constraints and the proportions of demand for different time-based service types can impact on inventory and transportation costs.

Under the EOQ inventory policy, the results show that distribution through a hierarchical organization of facilities, where customers are not necessarily served from the nearest stocking point (in order to allow a higher level of centralization while adhering to service time commitments), can lower the overall inventory levels. However, this is at the expense of increased transportation cost. Inventory cost decreases while transportation cost increases with the increase in the demand proportion for the relaxed service. In contrast, a non-hierarchical organization of service facilities, in which all customers are served from the nearest stocking point, can result in a lower average distance to reach customers at the expense of higher inventory cost. Also, as a non-hierarchical system treats all service calls in a similar fashion, overall similar inventory and transportation costs are incurred in serving customers with different service time requirements. The analysis shows that it can be beneficial to deploy stocks with high inventory related costs in a hierarchical fashion, while deploying stocks with low inventory related costs in a non-hierarchical fashion.

The investigation based on the (S-1, S) inventory policy gives some counterintuitive outcomes. Besides being the costlier option in terms of transportation, a hierarchical setup might not necessarily result in a lower total base-stock level compared to a non-hierarchical setup. Under the hierarchical setup, due to the discrete nature of the (S-1, S) inventory policy, inventory levels change stepwise when the demand proportions of the service types change; in several cases exceeding the required inventory level under the non-hierarchical setup. In such cases it can be financially better to distribute under a more decentralized (and hence more responsive) system as both inventory and transportation levels in the system can be lower.

It is important to emphasise the limitations of this research approach. Many of the assumptions in this work are not strictly appropriate in real life settings.

Clearly it cannot be claimed that the presented model, based on a stylized system, can be applied directly to a real world system. However, the model provides a non-complex tool for analysing distribution cost under constrained centralization levels. The analysis generates important generic insights which may not be generated through studying specific real life setups or instances. The insights can be useful to understand the likely impacts of different scenarios in time-differentiated distribution and can aid decision-making. The research could be extended in several ways, for example by considering inventory sharing in overlapping service ranges, service of multiple customers in one trip, and fill-rate based differentiation.

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