

# Retailer replenishment policies with one-way consumer-based substitution to increase profit and reduce food waste

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Received: 18 October 2018 / Accepted: 12 May 2020 / Published online: 27 May 2020  
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## ABSTRACT

Retailers can exploit the consumer willingness to substitute to improve their profit, service level and waste. This paper investigates to what extent such improvement can be realised by the replenishment decisions. Two order policies are compared: one policy neglecting product substitution, and a new policy that decides on order quantities for all products simultaneously meanwhile anticipating stock-out-based substitution.

Both policies are analysed by simulation-based optimisation. Besides finding the optimal parameter values or a variety of settings by exact enumeration (as a benchmark), we present for the case of one-way substitution a heuristic search procedure. The heuristic finds (nearly) optimal parameter values quickly and turns out to find optimal parameter values in almost all settings. An average profit increase of almost 9% is obtained when anticipating on substitution, while waste levels can decrease with more than 35%. A clear trade-off between service levels and profit/waste levels is found.

Assuming the retailer aims at profit maximisation, the service level of one product maybe very low or even zero. The results provide the following managerial insights in: (i) the service levels and waste levels that maximize the retailer's profit, (ii) whether a product should be removed from the assortment, (iii) the profit loss and waste increase of setting a higher (sub optimal) service level, e.g. for strategic reasons. Reversely, one may learn from the results what the profit margin of a product should be to justify a certain service level to a profit maximizing retailer. These insights maybe useful to retailers whose primary objective is beyond profit maximisation.

**KEYWORDS:** Retail · Food waste reduction · Substitution · Perishable · Multi-product



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## 1. INTRODUCTION

Food waste at retailers is both an economical and an environmental issue and should thus be prevented [1]. In this paper, we analyse an innovative way of reducing food waste at the retailer, by using a replenishment policy that incorporates substitution behaviour of the consumer.

Retailers sell many different products in their shop and for each individual item, replenishment decisions have to be taken. These decisions can either be done manually or are supported by computer aided ordering (CAO) or automated store ordering (ASO) systems. However, both approaches usually focus on individual products and therefore do not consider product substitution. At best, product substitution is anticipated informally in practice by setting different service levels for different products belonging to the same product category. Commonly used service levels are the in-stock probability and the fill rate. The impact of (differentiated) service levels on retailer profit and waste is not (a priori) known. As the main objective of

a retailer is profit maximisation, retailers are interested in a way to set replenishment quantities that maximise their profits. However, profit maximisation is not the only retail target. Most of them aim at high customer satisfaction and/or high market shares.

Retailers want to serve consumers at any time of the day and thus have the tendency to hold high inventory levels for every product in practice. For non-perishable products this is acceptable, as unsold goods can be sold later on. However, for perishable products, this strategy will lead to high waste levels due to product spoilage. When retailers re-think their strategy and accept out-of-stock situations for some products, while offering consumers a substitute product, inventory levels can be lowered and thus waste levels can be decreased. Research shows that consumers do accept substitute products in out-of-stock situations, although customer satisfaction might decrease [2]. The focus of this paper is on improving the replenishment decisions in a two perishable product situation, where the products are partly substitutes in case of an out-of-stock situation. According to [3], the willingness to substitute is for highly perishable products larger than for non-perishable products. Reasons for consumers to consider substitution are an out-of-stock situation of the preferred product, or a better value-for-money of a substitute product.

Price based substitution (as in [4]) is not considered in this paper. Neither do we consider quality or age-based substitution (e.g., see [5]). It has been shown in previous research [e.g. 6, 7, 8] that incorporating stock-out-based substitution in the replenishment decisions, can increase profit. However, it is not yet fully clear to what extent the trade-off between profit, waste and service levels are affected. As these other aspects are also of importance in retail, we optimise and compare two policies and report profit, waste, and  $\beta$ -service levels in this paper. We consider the fill rate (or  $\beta$ -service level) to be an appropriate service level definition in this context, as it indicates the fraction of demand that is lost or met by a substitute, which is more informative than the stock-out probability ( $\alpha$ -service level). First, we optimise for each product a base stock policy using independently a single product model, that thus does not include product substitution. Next, we optimise the replenishment parameters simultaneously using a multi-product model that includes product substitution. We use a multi-product simulation model to compare both approaches and report profit, waste, and  $\beta$ -service levels.

The objective of this paper is three-fold: (i) to present an approach to exploit product substitution in replenishment decisions, (ii) to generate managerial insights in the effect of product substitution on profit, waste and service levels, and (iii) to present a heuristic that facilitates the (heuristic) search for good replenishment parameters.

The remainder of this paper is organised as follow. In Section 2 we discuss the relevant literature on inventory

systems dealing with substitution and describe our contribution to the literature. Using a simulation-based optimisation approach, which is presented in Section 3 and the heuristic explained in Section 4. 576 instances are solved, and some managerial insights are discussed in Section 5. The paper finishes with conclusions and a discussion in Section 6.

## 2. LITERATURE REVIEW

Our study considers the replenishment of two products, of which one of them serves as substitute when the other product is out-of-stock. Although, the incorporation of substitution in replenishment decisions for perishable products is highly relevant for the grocery retail sector, the number of studies in the literature on this topic is limited [9]. In contrast, there are many studies dealing with substitution questions and non-perishable products [10]. One of the general conclusions of these papers is that the incorporation of substitution behaviour of consumers in the inventory decisions lead to better performances of the retailer. Therefore, it is very important to further study the incorporation of substitution behaviour for perishable products as well.

In this section, we will first discuss the studies on perishable products and continue with a short discussion on the studies for non-perishable products.

### 2.1. Replenishment decisions for perishable products

To our knowledge, only a few studies exist which include substitution in the replenishment decision for perishable products. One of them is the analytical study of Deniz et al. [11], where the replenishment is optimised for a product with a shelf life of 2 periods. The model is in fact a single product model with two demand classes related to the two age classes. In case of a stock out of one age class demand maybe substitute to another age class. Under the assumption of zero lead time, the problem is tractable, and an analytical solution is found. In a similar setting of a single product with two age classes, Sainathan integrates the replenishment and the pricing decision in [12]. Optimal solutions are obtained by applying the framework of Markov decision processes (MDP). The numerical determination of an optimal policy requires the state description of MDP to be low dimensional, such that the total number of states to evaluate is not too large. The state of a perishable inventory system is the number of products in stock in each age class, hence it is a vector. For all possible values of the state vector, a relative state value is determined as well as an optimal action. As the number of states increases rapidly with the dimension of the state vector, extending a single product MDP model to a two or multi-product model could make the model intractable. Similarly, an MDP solution cannot be found when the shelf life of the product gets too large [13, 14].

From the studies on perishable products, quite a few do not include consumer driven substitution, but supplier driven substitution. In this concept, the supplier decides which products will be issued and thus decides on product substitution. In [15] optimal issuing policies for perishable products are investigated for a single product with multiple demand classes using MDP. A retail example of supplier-based substitution is found in Chen et al. [6]. For perishable products, most research with supplier driven substitution is found in the context of blood banks. The distinction of blood types makes this setting a true multi-product setting, where the customers (medicines) set the substitution matrix based on the blood group of their patient and the compatibility of blood groups [16]. The blood banks aim at issuing a product from the same blood group as that of the patient, but it may decide to issue a substitute, if stock levels at blood banks require. Hajjema et al. [16] uses simulation-based optimisation which they combine with MDP. Duan and Liao [17] also applies simulation-based optimisation approach using tabu search and simulated annealing. Other examples in the blood supply chain are Dillon et al. [18] and Najafi et al. [19].

Duong et al. [20] concludes that studies on perishable inventories with substitution are scarce, while the context is very relevant to practice. Newsvendor models are appealing for imposing a structure that allows for mathematical analysis using renewal theory. Nevertheless, they propose a simulation approach by arguing that an exact method or a Newsvendor model is too limited, as the inventory dynamics become more complex when dealing with a longer shelf life, a positive lead time, and a lost sales context. An exact method becomes intractable for most settings in practice.

Besides these modelling and optimisation papers, it is worthwhile to mention the (more) empirical study of Sachs [8] and Kök and Fischer [21]. Both studies analyse the substitution behaviour of consumers based on sales data for perishable and non-perishable goods.

## 2.2. Replenishment decisions for non-perishable products

As by far most studies on replenishment decisions and product substitution concern non-perishable products, it is of interest to summarise the methods employed the obtained results in these settings and discuss whether the methods and results can be applied to perishables. According to the review of Shin et al. [10], many studies on consumer driven, inventory-based substitution are variations on the classical Newsvendor problem. Several studies do find optimal solutions analytically, like the study of Gaur and Honhon [22], Mahajan and v. Ryzin [23], Nagarajan and Rajagopalan [24], Netessine and Rudi [25], and Transchel [26]. A common approach is to model the problem as a single period problem and apply renewal theory. To some extent such approaches can be applied to perishable products, as discussed in the previous subsection. To obtain analytically a

closed form expression, several assumptions are put in place: e.g. zero lead time, backlogging, a short shelf life, or a replenishment and disposal policy that supports renewal point. An important difference to our study is the tractability of the problem. When adding multiple time periods, the whole problem becomes more complex and is not tractable anymore. Thus, it is not possible anymore to find an optimal solution analytically. Moreover, in order to obtain analytical solutions, traditionally one assumes zero lead time and backlogging to facilitate renewal theory. We will relax on these assumptions.

Dynamic programming methods are also present in literature (e.g. [27]), which may provide analytical results, but only when imposing strong assumptions similar to the Newsvendor models. A numerical solution of dynamic programming models allows relaxed assumptions (such as a positive lead time, lost sales, etc.) and is applicable to settings with at most a few (non-perishable) products, but when extending to perishable with a maximal shelf life, the state space of perishable inventory systems becomes too large to determine an optimal solution (similar to an MDP approach). A separate category of research on inventory pooling, where a stock out at a stock point is resolved by issuing demand from another stock point. As the products themselves are no different, we skip a discussing on inventory pooling models.

## 2.3. Contribution

From the above discussion it becomes clear that consumer-driven product substitution is hardly included in existing studies on the replenishment of perishable products, and vice versa multiperiod perishability is hardly included in existing models on product substitution. The main contribution of this paper is at that intersection: including perishability and product substitution in the replenishment of multiple perishable products. As the inventory dynamics is complicated by the perishability, positive lead time, and the substitution between two products, we adopt a simulation-based optimisation approach, which allows a greater modelling flexibility than the news vendor or single period models available in literature on product substitution. This methodology facilitates the evaluation of profit, waste, and service levels, all very important key performance indicators for a retailer. or the contribution in managerial insights we refer to Section 5.

## 3. METHOD

To analyse the effect of including substitution in replenishment decisions, a simulation-based optimisation model has been developed. The simulation model provides a good representation of the variability in the system, such as the demand and substitution uncertainty, and an accurate view of its effects

[28]. The effect of including substitution behaviour can be measured in terms of profit increase, waste decrease or obtaining better service levels. To obtain an understanding of the influencing factors, several parameters are analysed such as the remaining shelf life, the substitution fraction, or the procurement costs. These parameters are further discussed in Section 5. This section continues with the problem description, followed by the notation and the mathematical model.

### 3.1. Problem description

We focus on a product category of a retailer that consists of  $N$  different products with a fixed maximum shelf life  $M_i$ . Within this product category, the retailer chooses a main product (from now on ‘product 1’) which serves as a substitute when other products of this product category are out-of-stock. The willingness of consumers to buy this substitute is given by a fraction ( $\gamma_j$ ). The retailer faces a Poisson demand for all  $N$  products meaning that consumers arrive at the rate of the Poisson distribution and request 1 item of the product. The retailer places an order at the beginning of the period, before opening hours. This order will arrive after closing, resulting in an effective lead-time of one day and an effective shelf life of  $M_i - 1$ .

### 3.2. Notation

Sets and indices:

$i, j \in \{1, \dots, N\}$	Products
$t \in \{1, \dots, T\}$	Time periods
$r, m \in \{1, \dots, M_i\}$	Remaining shelf life

Parameters:

$S_i$	Order-up-to level of product $i$
$\mu_i$	Mean demand of product $i$
$\gamma_{ij}$	Substitution fraction of product $i$ to product $j$
$p_i$	Sales price of product $i$
$c_i$	Cost of product $i$
$a$	Fraction FIFO consumers
$M_i$	Maximum shelf life of product $i$ upon arrival at the retailer

Variables:

$I_{tir}$	Inventory of product $i$ at the beginning of time period $t$ with remaining shelf life $r$
$D_{ti}$	Initial demand of product $i$ during time period $t$
$EO_{ti}$	Estimated outdating of product $i$ at time period $t$
$Q_{ti}$	Order quantity of product $i$ at time period $t$
$X_{ti}$	Remaining demand of product $i$ that should be fulfilled by a substitute at time period $t$
$Z_{tir}$	Products sold of product $i$ at time period $t$ with remaining shelf life $r$ without substitution
$U_{tir}$	Products sold of product $i$ at time period $t$ with remaining shelf life $r$ due to substitution
$W_{ti}$	Waste of product $i$ at the end of time period $t$
$\Pi_t$	Total profit of time period $t$

$D_{ti}, X_{ti}, Z_{tir}, U_{tir}$  will be split into FIFO ( $DF_{ti}, XF_{ti}, ZF_{tir}, UF_{tir}$ ) and LIFO ( $DL_{ti}, XL_{ti}, ZL_{tir}, UL_{tir}$ ), see the model below.

### 3.3. Discrete time simulation model

At the beginning of every period, the retailer places an order  $Q_{ti}$  for all  $N$  products, see equation (1), based on the order-up-to level ( $S_i$ ), the current inventory for product  $i$  and the estimated outdating during that period. Outdating is estimated by the difference

$$Q_{ti} = [S_i - \sum_{r=1}^{M_i-1} I_{tir} + EO_{ti}]^+$$

$$EO_{ti} = [I_{ti1} - (\mu_i * a)]^+$$

between the average FIFO demand per day and the current inventory with a remaining shelf life of 1 day, equation (2). This approach is taken for practical reasons. However, it might lead to an overestimation of the outdating.

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (1)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (2)$$

Demand for each product follows a Poisson distribution with mean  $\mu_i$ , equation (3). At a retailer, there are usually consumers who prefer the fresher products, and some that are more indifferent with respect to age and thus tend to take the older products.

$$D_{ti} \sim Poiss(\mu_i)$$

$$DF_{ti} = round(a * D_{ti})$$

$$DL_{ti} = D_{ti} - DF_{ti}$$

Therefore, we split the total demand for the products into a FIFO and LIFO demand equation (4) and equation (5), with  $a$  being the fraction of demand following FIFO withdrawal.

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (3)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (4)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (5)$$

The product withdrawal by consumers for both LIFO and FIFO demand is the minimum of the products in stock of a certain age  $r$  and the remaining demand that is not satisfied yet. Without loss of generality, it is

$$ZL_{tir} = \min\{I_{tir}, DL_{ti} - \sum_{m=r+1}^{M_i} ZL_{tim}\} \quad (6)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, r \in \{1, \dots, M_i - 1\}$$

$$ZF_{tir} = \min\{I_{tir} - ZL_{tir}, DF_{ti} - \sum_{m=1}^{r-1} ZF_{tim}\} \quad (7)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, r \in \{1, \dots, M_i\}$$

When the demand is not met, substitution might take place. The number of consumers requesting a substitute product is an average fraction ( $\gamma_{ij}$ ) of the

assumed that customers preferring the freshest product arrive first at the supermarket, and thus LIFO demand is fulfilled before the FIFO demand.

consumers facing a stock-out and given by equation (8), with  $Z_{tir}$  being the total demand which is already met (equation (9))

$$X_{ti} \sim \text{Binom}(D_{ti} - \sum_{r=1}^{M-1} Z_{tir}, \gamma_{ij}) \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (8)$$

$$Z_{tir} = ZL_{tir} + ZF_{tir} \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, r \in \{1, \dots, M_i - 1\} \quad (9)$$

Similar to the initial demand, the demand arising due to substitution is also divided into LIFO and FIFO demand:

$$XF_{ti} = \text{round}(a * X_{ti}) \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (10)$$

$$XL_{ti} = X_{ti} - XF_{ti} \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (11)$$

As it is assumed that stock-outs are more likely to happen at the end of a day, the demand occurring due to substitution takes place after the initial demand of

the product is fulfilled, first by the LIFO withdrawal, followed by the FIFO withdrawal.

$$UL_{tir} = \min\{I_{tir} - Z_{tir}, \sum_{j \neq i} XL_{tj} - \sum_{m=r+1}^{M_i} UL_{tim}\} \quad (12)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, r \in \{1, \dots, M_i - 1\}$$

$$UF_{tir} = \min\{I_{tir} - Z_{tir} - UL_{tir}, \sum_{j \neq i} XF_{tj} - \sum_{m=1}^{r-1} UF_{tim}\} \quad (13)$$

$$\forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, r \in \{1, \dots, M_i\}$$

At the end of a period, the inventory is updated for the next period, the shelf life is reduced with one period and outdating is registered, consumer withdrawal is subtracted and the products ordered at the beginning

of the day will arrive, equation (14), with  $U_{tir}$  being the total demand fulfilled by substitution (equation (15)). Note: the effective shelf life of a product is  $M - 1$ . A lead-time of 1 day ( $L = 1$ ) is applied for the retailer.

$$I_{t+1,n,r-1} = \begin{cases} I_{tir} - Z_{tir} - U_{tir} & , 2 \leq r < M_i - 1 \\ Q_{ti} & , r = M_i \end{cases} \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (14)$$

$$U_{tir} = UL_{tir} + UF_{tir} \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, r \in \{1, \dots, M_i - 1\} \quad (15)$$

### 3.4. Performance indicators

To analyse the performance of a retailer, several key performance indicators are used. To determine the optimal order-up-to level  $S$ , total profit is maximised. Profit is calculated by subtracting procurement costs from the revenue made (equation (16)) and reported as daily profit (equation (17)). Fixed ordering costs and

holding costs are neglected as perishable products at a retailer are usually replenished daily together with many others and therefore ordering and transportation costs are shared among all those products [29]. Excessive inventory levels do not occur, as the shelf life is short.

$$\Pi_t = \sum_{i=1}^N (p_i * (\sum_{r=1}^{R-1} Z_{tir} + U_{tir}) - Q_{ti} * c_i) \quad \forall t \in \{1, \dots, T\} \quad (16)$$

$$\Pi = \frac{\sum_{t=w+1}^T \Pi_t}{T - w} \quad (17)$$

Waste is calculated for every product per period of time by equation (18). For the final analysis, waste is

represented as percentage of total ordered products, equation (19).

$$W_{ti} = I_{ti1} - Z_{ti1} - U_{ti1} \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\} \quad (18)$$

$$W = \frac{\sum_{t=w+1}^T \sum_{i=1}^N W_{ti}}{\sum_{t=w+1}^T \sum_{i=1}^N Q_{ti}} \quad (19)$$

Moreover, service level measures are included. The fraction of demand that can be fulfilled from stock for the non-substitute product is measured by the  $\beta$ -service level. The  $\beta_i$ -service level represents the fraction of fulfilled demand for product  $i$ , equation (20). For the product which remaining demand is fulfilled by the other product, the  $\beta_j$ -service level measures the fraction of fulfilled demand for product  $j$ , either by product  $j$ , or

product  $i$ , equation (21), with  $j \neq i$ . To specify by which products this demand is fulfilled, we included the  $\beta_{ji}$ - and  $\beta_{jj}$ -service level as well. The  $\beta_{ji}$ -service level is the fraction of demand for product  $j$  fulfilled by product  $i$ , equation (22), and the  $\beta_{jj}$ -service level is the fraction of remaining demand for product  $j$  fulfilled by product  $j$ , equation (23). Those fractions are estimated, per day, in the simulation as:

$$\beta_i = \frac{\sum_{t=w+1}^T \sum_{r=1}^{R-1} Z_{tir}}{\sum_{t=w+1}^T D_{ti}} \quad \forall i \in \{1, \dots, N\} \quad (20)$$

$$\beta_j = \frac{\sum_{t=w+1}^T \sum_{r=1}^{R-1} Z_{tjr} + U_{tir}}{\sum_{t=w+1}^T D_{tj}} \quad \forall j \in \{1, \dots, N\}, j \neq i \quad (21)$$

$$\beta_{ji} = \frac{\sum_{t=w+1}^T \sum_{r=1}^{R-1} U_{tir}}{\sum_{t=w+1}^T X_{tj}} \quad \forall i \in \{1, \dots, N\}, j \in \{1, \dots, N\}, j \neq i \quad (22)$$

$$\beta_{jj} = \frac{\sum_{t=w+1}^T \sum_{r=1}^{R-1} Z_{tjr}}{\sum_{t=w+1}^T D_{tj}} \quad \forall j \in \{1, \dots, N\} \quad (23)$$

### 3.5. Optimisation approach

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**Algorithm:**


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- IN:  $\gamma, \mu, p_i, c_i, M$   
 OUT:  $S_1^*, S_2^*, \Pi^*, \Delta\Pi, \text{Waste}, \Delta W$  and  $\beta$ -service levels
- 1: Determine individual order-up-to levels  $\hat{S}_1$  and  $\hat{S}_2$  by full enumeration, with the simulation model without substitution.
  - 2: Evaluate  $\hat{S}_1$  and  $\hat{S}_2$  in setting with the simulation model including substitution behaviour of the consumer to find corresponding profit ( $\hat{\Pi}$ ) and waste ( $\hat{W}$ ) levels.
  - 3: Determine optimal order-up-to levels  $S_1^*, S_2^*$  with substitution based ordering by full enumeration, with the substitution model.
  - 4: Compare results obtained at step 2 with results of step 3
- 

The optimisation algorithm consists of multiple steps. First the optimal order-up-to level  $S$  for a single product is determined with the help of the simulation model. This is done for each of the  $N$  products individually, and thus substitution is not incorporated yet. The optimal order-up-to levels are determined based on profit maximisation and their values are denoted  $\hat{S}_i$ . In the second step, the order-up-to levels  $\hat{S}_i$  found in step 1 are used as reference values and therefore the simulation model is ran with these order-up-to levels when substitution does play a role. In the third step of the optimisation, every order-up-to level combination for the  $N$  products is evaluated and the optimal order-up-to levels  $S_i^*$  are determined. For the final analysis, the values obtained in step 2 are compared with the values obtained in step 3. Both in step 1 and in step 3 of the optimisation a full enumeration is performed over a range of order-up-to levels  $S$  ( $\{S_i^{min}, \dots, S_i^{max}\}$ ).

The lower and upper level  $S_i^{min}, S_i^{max}$  of the search range are determined as follow. For Poisson demand, the order-up-to level  $S$  can be calculated based on the lead-time ( $L$ ), review period ( $R$ ), the average demand ( $\mu$ ) and the safety factor ( $z$ ), using equation (24). In this research, the lead-time and review period are both fixed to 1 period.

$$S = \mu_{R+L} + z * \sqrt{\sigma_{R+L}} \quad (24)$$

To calculate the order-up-to level  $S_i^{max}$ , the demand for both products is combined (thus,  $\mu = \mu_1 + \mu_2$ ). A lower bound ( $S_i^{min}$ ) of 0 is used as it might be beneficial not to have a product in stock at all, or to have a negative safety stock.

For the Base Case scenario, also a minimisation on waste has been performed. The maximum waste reduction is determined without profit losses, compared to optimisation without anticipating substitution.

We also investigated a policy that considers the combined age distribution applying Stochastic Dynamic Programming [13]. It appeared that the improvement over an order up to policy for our experiments is less than 1% for very perishable products and nearly absent when the shelf life is larger than 4. Therefore, we conclude that the easier to implement order-up-to

policy is quite robust with respect to the optimal profit that can be reached.

#### 4. HEURISTIC TO FIND (NEAR) OPTIMAL ORDER-UP-TO LEVELS

Optimising replenishment for perishable products is a complex task, due to all the interdependencies between the products. Thus, the complete enumeration is computationally expensive. By developing a heuristic, the runtime needed to find a solution decrease. The heuristic developed in this study still includes the simulation model described in section 3, and is therefore able to deal with a lead-time larger than zero and the perishability of the products. For notational convenience we present the heuristic for two products, but it can be extended to more than two products. Product 1 is the main product to which substitution may take place.

The heuristic is based on interesting characteristics of the results of Section 5, found by complete enumeration. For every experiment, the optimal  $S_1^*$ -level is higher than or equal to the optimal  $\hat{S}_1$ -level. For the product not serving as a substitute, the exact opposite characteristic is valid. For every experiment, the optimal  $S_2^*$ -level was equal to or lower than the optimal  $\hat{S}_2$ -level. These structural properties can be used to improve the optimisation process, as many possible combinations of  $S_1$  and  $S_2$  will never be optimal. Thus, these combinations can be excluded.

The developed approach consists of several steps. In the first step, the individual order-up-to levels  $\hat{S}_1$  and  $\hat{S}_2$  are calculated with full enumeration over all order-up-to levels  $S$ . The found order-up-to levels serve as starting point for the rest of the approach. Two starting points are used, (i)  $S_1 = \hat{S}_1$  and  $S_2 = \hat{S}_2$  (resulting in  $\bar{\Pi}_1$ ) and (ii)  $S_1 = \hat{S}_1 + \hat{S}_2$  and  $S_2 = 0$  (resulting in  $\bar{\Pi}_2$ ). Then, based on which of the two options results in the highest profit, the heuristic continues with another step. When  $\bar{\Pi}_1 > \bar{\Pi}_2$ , the heuristic continues with step 3, otherwise it continues at step 4. The third step consists of two parts. First, we keep the total inventory level ( $S_1 + S_2$ ) the same, but increase  $S_1$  and decrease  $S_2$  by 1

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**Heuristic**

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IN:  $\gamma, \mu, p_i, c_i, M$

OUT:  $S_1^*, S_2^*, \Pi^*$  (and Waste and  $\beta$ -service levels)

1: Determine individual order-up-to levels  $\hat{S}_1$  and  $\hat{S}_2$

2: Set  $\hat{\Pi}$  with  $S_1 = \hat{S}_1$  and  $S_2 = \hat{S}_2$  and determine  $\Pi_x$  with  $S_1 = \hat{S}_1 + \hat{S}_2$  and  $S_2 = 0$  and determine highest profit. If  $\hat{\Pi} \geq \Pi_x$  continue with step 3, else continue with step 4

3.1: Iteratively increase  $S_1$  ( $S_1 + 1$ ) and decrease  $S_2$  ( $S_2 - 1$ ) until best solution is found in terms of profit. Update  $S_1$  and  $S_2$

3.2: Iteratively check neighbourhood  $\{(S_1 + 1, S_2)/(S_1 - 1, S_2)/(S_1, S_2 + 1)/(S_1, S_2 - 1)\}$  of best solution found so far until no better solution is found. Update  $S_1$  and  $S_2$ . STOP

4: Iteratively in-/decrease  $S_1$  until no better solution is found. Update  $S_1$ , keep  $S_2 = 0$ . STOP

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unit. This is iteratively done until no better solution is found for 3 consecutive runs. Then the neighbourhood of the best solution is checked, to see if a better solution exists, by either fixing  $S_1$  or  $S_2$  and in-/decreasing the other by 1 unit. When the best solution is found, and 3 consecutive runs do not give a better solution, the search stops. When step 4 is applied,  $S_2$  is always equal to 0, and  $S_1$  is iteratively in-/decreased by 1 unit until no better solution is found (in terms of profit) for 3 consecutive runs. Then the optimal solution is found, and the procedure stops.

## 5. NUMERICAL RESULTS

In this section, we first discuss the experimental design. Next, we provide an overview of results and discuss the benefits of exploiting product substitution in the replenishment decision. We will zoom into the base case, and derive managerial insights, e.g. on profit maximisation versus waste minimisation (with a profit constraint), on the assortment decision, and on sub optimal service levels and profit margins. Finally, we discuss the (nearly optimal) performance of the heuristic.

### 5.1. Experimental design

The base case studied in this section is a setting where two highly perishable products ( $N = 2$ ) are considered. We assume the demand to be Poisson distributed with

identical means for both products:  $\mu_1 = \mu_2 = 5$ . The shelf life is set to three ( $M_i = 3$ ), and the procurement costs are set to  $c_1 = c_2 = \text{€}0.5$ . The selling price is fixed to ( $p_i$ ) of  $\text{€}1$  for both products, thus resulting in a profit margin of  $\frac{p_i - c_i}{p_i} = 50\%$ , which is realistic for many perishable grocery products, like packed meat and fresh cut lettuce. The symmetry in the base case between the products gives a good understanding about the effect of substitution behaviour that will not be influenced by other parameters. In the base case, we set  $a = 0.5$ , i.e. 50% of the consumers are of the FIFO type (accepting the oldest available product), the other 50% select the freshest available products. To calculate the upper order-up-to level  $S_i^{max}$ , used in the heuristic, a safety factor  $z$  equal to 3 is used for every experiment. For the given a Poisson distribution for the demand would result in a service level of 99.99%. Moreover, a 100% service level would result very high waste levels and thus low profit levels and would be an unrealistic target for the retailer.

There are multiple factors influencing the performance of the retailer. An obvious one is the demand per product. Besides an equal demand per product, it is analysed how a different demand per product influences the results. Moreover, different procurement costs can lead to a different optimal solution. Furthermore, we expect the shelf life of the product to be of great influence on the retailer performance, as a longer shelf life gives more time to sell the product instead of waste them [29]. Therefore,

Table 1: Experimental values for the 576 experiments

Factor	Notation	Values
Substitution rate	$\gamma_{21}$	$\in \{0.5, 0.75, 0.9, 1\}$
Mean demands	$(\mu_1, \mu_2)$	$\in \{(5,5), (3,7), (7,3)\}$
Shelf life	$(M_1, M_2)$	$\in \{(3,3),(5,5),(3, 5),(5,3)\}$
Procurement costs	$(c_1, c_2)$	$\in \{(0.5,0.5), (0.7,0.7), (0.5,0.7), (0.7,0.5)\}$
Fraction FIFO consumers	$a$	$\in \{0, 0.5, 1\}$



the effect of shelf life is analysed, both in terms of an extended shelf life, as in terms of an unequal shelf life for the two products. The complete overview of the experimental factor values induced is given in Table 1. The settings chosen for the experiment resemble realistic values for highly perishable products at the retailer such as lettuce and meat [29, 30].

A full factorial design is used, resulting in 576 experiments. The model is implemented in Matlab2018a. A run length of  $T = 10000$  periods is applied, which including a warming up of  $w=20$  periods, to ensure the performance measures are accurately evaluated. All experiments are executed 20 times with different (sampled) demand data sets. To give an impression of the accuracy level achieved: for the base case, the 90% confidence interval of the mean profit per period is [€4.321 – €4.328].

## 5.2. Overview: effect of substitution-based ordering

An overview of the impact of exploiting product substitution in the replenishment decision is given in Table 2. For every factor, the number of experiments is given, the average change in profit and waste is reported when moving from traditional independent ordering with order up to levels  $\hat{S}$  to the new policy that anticipates exploit product substitution (with optimal levels  $S_i^*$ ), as well as the (profit maximising)  $\beta$ -service levels.

The following tendency can be observed using individual order-up-to levels  $\hat{S}_i$  compared to substitution-based order-up-to levels  $S_i^*$ . When many consumers are willing to buy a substitute, it becomes more beneficial to have higher stock levels of the product that serves as substitute (e.g. product 1) and less of the other product (product 2). When basically everyone is accepting a substitute, the largest profit increase will be obtained when only product 1 is in stock compared to keep both product 1 and 2 in stock. Furthermore, waste levels can be reduced drastically. As all consumers accept a substitute, demand for both products can be combined which reduces the relative variation, like is the case with inventory pooling [31]. This facilitates a better determination of the optimal order-up-to levels  $S$  and leads to an increase in profit and decrease in waste. When less consumers are willing to buy a substitute, the optimal order-up-to levels  $S^*$  will be lower for  $S_1$  and higher for  $S_2$ . Due to consumers that are not willing to buy a substitute, it is necessary to keep both products in stock to maintain the sales. Otherwise, the amount of lost sales will be too high and profit decreases. The waste reduction therefore decreases, as both products must be kept in stock, although substitution-based ordering still improves compared to independent ordering.

The  $\beta_1$ - and  $\beta_2$ -service levels are for every case high and thus most consumers will leave the store with a product. The  $\beta_{22}$ -service level shows which fraction of the demand for product 2 is also fulfilled by product 2,

where the  $\beta_{21}$ -service level indicates which fraction of the demand is fulfilled by product 1. When the level of substitution is high, and thus  $S_2$  is low in many cases, demand for product 2 is fulfilled by product 1, reducing the  $\beta_{22}$ -service level. Moreover, the  $\beta_{21}$ -service level increases with lower substitution rate  $\gamma_{21}$ . Thus, a higher percentage of consumers that want to buy a substitute will find a product in the shelf.

When the demand of the two products is not equal, and the product not serving as a substitute has a higher demand than the substitute, a larger profit increase is obtained when substitution-based ordering is applied. However, a larger waste reduction is obtained when the substitute product has the highest average demand. In absolute figures, waste is lower for products with a longer shelf life. As the optimal order-up-to levels  $S$  change, a change in the  $\beta$ -service level is obtained.

The effect of substitution is more profound when the shelf life of the products is slightly increased. With a maximum shelf life of 5 days, the obtained profit increase is larger compared to a maximum shelf life of 3 days. Moreover, a larger waste reduction is obtained for the case of a longer shelf life. Service levels are also higher for a longer shelf life, as it is less complicated to keep the right number of products in stock. This can also be seen in the results of a different shelf life for both products. When the shelf life of product 1 is lower than of product 2, hardly any profit increase or waste reduction is obtained. As a longer shelf life allows to have a higher quantity in stock, product 2 should be still available. When the shelf life of product 2 is lower than of product 1, there is a large reduction in waste levels possible. The reduction in the  $\beta_{22}$ -service level indicates the reduced inventory level of the product.

Interesting results are obtained when the ratio between procurement cost and sales price differs. The best profit increase is found when product 2 is less profitable than the substitute (product 1). In this case, it is beneficial to keep mainly the substitute in stock, as consumers will buy a more profitable product in the end. The high stock level of  $S_1$  also results in a high  $\beta_1$ -service level. When the substitute product is the least profitable product, it is beneficial to avoid out-of-stock situations of the non-substitute product and thus keep both products in stock. Therefore, the possible profit increase is small. When both products have a similar but high procurement cost, it becomes more expensive to have waste. Therefore, both products will have a reduced optimal order-up-to level  $S$  and thus a reduction in  $\beta$ -service levels is present. Although, anticipating on substitution still results in a profit increase.

With a full FIFO withdrawal, the highest waste reductions can be obtained. As a FIFO consumer withdrawal of the products is the most efficient in terms of waste, which leads to the potential improvements. On the other hand, the profit increases are not very high. With a full LIFO withdrawal, high profit increases can be obtained by the combined replenishment

Table 2: Average results of all experiments and datasets listed per experimental factor. Number of experiments per dataset (#), relative change in profit ( $\Delta\Pi = \frac{\Pi^* - \hat{\Pi}}{\hat{\Pi}}$ ) and waste ( $\Delta W = \frac{W^* - \hat{W}}{\hat{W}}$ ) and  $\beta$ -service levels for the optimal order-up-to levels  $S^*$ .

Dataset	#	$\Delta\Pi$	$\Delta W$	$\beta_1$	$\beta_2$	$\beta_{22}$	$\beta_{21}$
All Experiments	576	8.89%	-35.27%	94.99%	89.38%	55.55%	93.70%
<b>Substitution fraction</b>							
$\gamma_{21}=0.5$	144	4.53%	-22.16%	93.56%	87.56%	76.48%	96.83%
$\gamma_{21}=0.75$	144	9.33%	-37.91%	95.35%	89.09%	53.02%	93.11%
$\gamma_{21}=0.9$	144	9.34%	-37.91%	95.35%	89.09%	53.02%	93.10%
$\gamma_{21}=1$	144	12.37%	-43.09%	95.72%	91.77%	39.71%	91.77%
<b>Average demand</b>							
$(\mu_1, \mu_2)=(5,5)$	192	9.34%	-35.52%	95.42%	89.77%	58.26%	94.22%
$(\mu_1, \mu_2)=(3,7)$	192	10.21%	-26.68%	93.24%	91.71%	67.92%	95.57%
$(\mu_1, \mu_2)=(7,3)$	192	7.13%	-43.60%	96.32%	86.66%	40.49%	91.31%
<b>Max. shelf life</b>							
$(M_1, M_2)=(3,3)$	144	7.29%	-17.72%	95.41%	84.69%	46.12%	90.62%
$(M_1, M_2)=(3,3)$	144	7.90%	-66.60%	97.33%	88.91%	44.33%	94.82%
$(M_1, M_2)=(3,3)$	144	4.13%	-2.42%	90.55%	90.96%	71.69%	93.60%
$(M_1, M_2)=(3,3)$	144	16.25%	-54.33%	96.69%	92.95%	60.08%	95.77%
<b>Price</b>							
$(p_1, p_2)=(0.5,0.5)$	144	5.67%	-40.36%	96.08%	93.11%	65.16%	95.45%
$(p_1, p_2)=(0.7,0.5)$	144	3.21%	-25.32%	91.39%	93.87%	84.19%	95.48%
$(p_1, p_2)=(0.5,0.7)$	144	20.74%	-45.55%	98.47%	82.58%	19.67%	92.20%
$(p_1, p_2)=(0.7,0.7)$	144	5.94%	-29.84%	94.03%	87.95%	53.20%	91.68%
<b>FIFO/LIFO</b>							
$a=0$	192	15.75%	-33.80%	91.04%	82.66%	60.10%	86.79%
$a=1$	192	4.81%	-40.25%	97.24%	94.41%	57.77%	98.07%
$a=0.5$	192	6.11%	-31.75%	96.71%	91.06%	48.79%	96.24%

strategy. The mixture of a 50% FIFO and a 50% LIFO withdrawal shows results most similar to the full FIFO withdrawal.

### 5.3. Trade-off between service levels and profit or waste

In retail, it might be interesting to set target service levels for the products, to satisfy consumers as much as possible. In the previous sections, we showed that, due to substitution effects, it could be more attractive to have only the product serving as substitute in stock instead of both products. A decision to also keep product 2 in stock implies a trade-off between profit

and service level, and between waste and the service level. Figure 1 depicts this trade-off for the base case by showing the performance for different values of  $S_2 \in \{0, \dots, 22\}$  (while keeping  $S_1$  to its optimal value). When the service level of product 2 ( $\beta_{22}$ ) becomes higher, the maximum daily profit reduces. There may be strategic reasons (e.g. service level or market share consideration) to require a minimum service level  $\beta_{22}$ . The curve shows the profit loss of setting a sub-optimal service level, and consequently it may help to derive the price of a certain service level as well as the profit margin associated with a certain service level optimal.

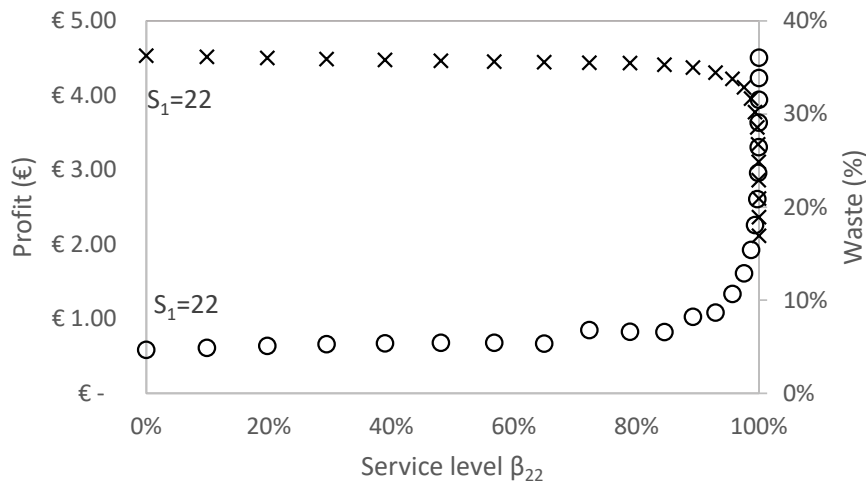


Figure 1: Total profit in €(x) and Waste in % (o) vs. service levels  $\beta_{22}$  for  $S_2 \in \{0, \dots, 22\}$  with full substitution ( $\gamma_{21} = 1$ )

Obviously, product waste minimisation and service levels maximisation are contradicting performance indicators: setting a high service levels commonly implies high product waste. Figure 1 illustrates the conflicting character of service level and profit objectives. On the one hand, the retailer might want to serve the customers always with the product consumers demand, where on the other hand they aim for high profit levels and/or low waste levels. Figure 1 shows that in case of full substitution, high profit levels and low waste levels are obtained when the service level of product 2 ( $\beta_{22}$ ) is not too high. Although, when a retailer is willing to compromise slightly on the service level (e.g.  $\beta_{22} = 80\%$ ), decent profits can be made together with low waste levels.

**5.4. Maximising profit and minimising waste for the Base Case**

In section 5.2, the focus of optimisation was on profit maximisation. Although a waste reduction was obtained for every experiment, larger waste reductions might be possible with different order-up-to levels  $S$ . In this section, we therefore analyse the Base Case both

on profit maximisation and on waste minimisation. When the focus of the optimisation is on waste minimisation, a profit constraint is added, to ensure profit does not decrease compared to the profit values of the individual ordering settings. Moreover, the more detailed results also give extra insight into the trade-off between service-levels and profit or waste. The experimental settings of the Base Case are as follows, the average demand  $\mu_1 = \mu_2 = 5$ , the procurement costs  $p_1 = p_2 = \text{€}0.5$  and the maximum shelf life  $M_i = 3$  for both products. Consumer withdrawal is considered a mixture between 50% FIFO and 50% LIFO.

Table 3 shows the optimal order-up-to levels  $S^*$ , the obtained profit and resulting waste (in percentages of total ordered quantity) and the various  $\beta$ -service levels are shown for the varying substitution behaviour  $\gamma_{21}$ . In these results the order-up-to levels  $S^*$  for substitution-based ordering. Furthermore, the in-/decrease in profit and waste is given. The differences are calculated by comparing order-up-to levels  $S_i^*$  (step 3 of the optimisation procedure) and individual order-up-to levels  $\hat{S}_1$  (step 2 of the optimisation procedure).

Table 3:  $S_1^*, S_2^*, \text{Profit } (\Pi), \text{Waste } (W), \beta\text{-service levels and relative change in profit } (\Delta\Pi = \frac{\Pi^* - \hat{\Pi}}{\hat{\Pi}})$  and waste ( $\Delta W = \frac{W^* - \hat{W}}{\hat{W}}$ ) compared to sub-optimal replenishment ( $\hat{S}$ ) when maximising profit.  $\hat{S}_1 = \hat{S}_2 = 12$

$\gamma_{21}$	$S_1^*$	$S_2^*$	$\Pi$	$\Delta \Pi$	$W$	$\Delta W$	$\beta_1$	$\beta_2$	$\beta_{22}$	$\beta_{21}$
0.5	13	10	€4.33	1.29%	7.11%	-19.73%	95.40%	91.69%	84.58%	97.52%
0.75	15	7	€4.44	3.04%	5.39%	-37.98%	97.46%	90.13%	64.94%	91.91%
0.9	15	7	€4.44	3.04%	5.39%	-37.98%	97.46%	90.13%	64.94%	91.91%
1	22	0	€4.53	5.28%	4.68%	-46.16%	99.84%	90.26%	0.00%	90.26%

Table 4:  $S_1^*$ ,  $S_2^*$ , Profit ( $\Pi$ ), Waste ( $W$ ),  $\beta$ -service levels and relative change in profit ( $\Delta\Pi = \frac{\Pi^* - \hat{\Pi}}{\hat{\Pi}}$ ) and waste ( $\Delta W = \frac{W^* - \hat{W}}{\hat{W}}$ ) compared to sub-optimal replenishment ( $\hat{S}$ ). Highest possible waste reduction without profit loss.  $\hat{S}_1 = \hat{S}_2 = 12$

$\gamma_{21}$	$S_1^*$	$S_2^*$	$\Pi$	$\Delta\Pi$	$W$	$\Delta W$	$\beta_1$	$\beta_2$	$\beta_{22}$	$\beta_{21}$
0.5	11	9	€4.21	1.12%	3.55%	-59.89%	88.58%	85.76%	78.95%	94.04%
0.75	19	1	€4.32	0.39%	2.82%	-67.55%	99.42%	78.15%	89.34%	9.90%
0.9	19	1	€4.32	0.39%	2.82%	-67.55%	99.42%	78.15%	89.34%	9.90%
1	19	0	€4.32	0.39%	1.49%	-82.81%	99.16%	75.71%	75.71%	0.00%

The results clearly have the same trend as shown in Table 2, when the willingness of consumers to substitute ( $\gamma$ ) increases, larger profit improvements are obtained. When all consumers are willing to substitute, it is the most profitable to have only product 1 in stock and divert all customers towards product 1. This results in a high  $\beta_1$ - and  $\beta_2$ -service level, however the  $\beta_{22}$ -service level is 0.00%, as product 2 is not available. At lower substitution rates  $\gamma$ , it becomes more beneficial to also have product 2 in stock. This increases the  $\beta_{22}$ -service level but also increases the waste and lower the profit increase.

Table 4 shows the optimal order-up-to levels  $S$  when waste is minimised, without decreasing the profit levels obtained with  $\hat{S}$ . The combined order-up-to levels  $S^*$  are lower for waste minimisation than for profit maximisation. This reduces the number of products that are sold; thus, the obtained profit is lower. Although the profit is reduced, profit levels for waste minimisation are still higher than the profit levels obtained at individual ordering. Moreover, the reduction of products in stock also reduces the number of products turning into waste at the end of their shelf life. Waste levels become significantly lower (e.g. 5.1% versus 1.8% for  $\gamma_{21} = 1$ ). Furthermore, the reduction in optimal order-up-to levels also decreases the service levels.

### 5.5. Performance of the heuristic

The developed heuristic is used for all 576 experiments listed in Section 5. In all cases, it led to the optimal solution found by full enumeration for both order-up-to levels  $S$ . At the second step of the procedure, a decision is made whether to continue with step 3 or step 4. In 75% of the cases,  $\bar{\Pi}_1 \geq \bar{\Pi}_2$  holds and thus step 3 is used. For all other cases (25%) the heuristic continues at step 4.

Table 5 shows that most improvement is obtained in the execution of the fourth step. As found in Table 2, it is sometimes optimal to have only product 1 in stock. This occurs mostly when substitution rates are high. Thus, the optimal solution will be found by step 4 in the heuristic. In the case where substitution rates are lower, the optimal solution for  $S_1^*$  and  $S_2^*$  will be much

Table 5: Performance of heuristic. Per step the average number of runs needed and the number of experiments using this step, the relative change in profit ( $\Delta\Pi = \frac{\Pi^* - \hat{\Pi}}{\hat{\Pi}}$ ) (in %).

Step	# runs	$\Delta\Pi$	# experiments
1	62	-	576
2	2	-	576
3.1	4.01	+1.57%	432
3.2	19.17	+5.76%	432
4	7.30	+12.98%	144
All	98.48	+6.77%	576

closer to  $\hat{S}_1$  and  $\hat{S}_2$  and therefore the best solution will be found by step 3 of the heuristic. As the optimal solution is close to  $\hat{S}_1$  and  $\hat{S}_2$ , the profit increase obtained is also lower.

The advantage of using the heuristic versus the full enumeration is the saving in number of simulation runs. When full enumeration is used to obtain the optimal order-up-to levels  $S_i^*$ , the simulation model is ran for 1023 times. First  $31 + 31 = 62$  times to determine  $\hat{S}_1$  and  $\hat{S}_2$ , followed by  $31 * 31 = 961$  times to determine the optimal order-up-to levels  $S_i^*$ . The number of runs needed by the heuristic to find the optimal solution is significantly lower, as shown in Table 5. In step 1 and 2 of the heuristic, still 64 runs are needed, but the number of runs needed to obtain optimal order-up-to levels  $S_i^*$ , in step 3 or 4 is reduced significantly. On average a 90.8% saving in the number of simulation runs is obtained. As a single simulation run (for one  $S_1/S_2$  combination) takes approximately 5.3 seconds, taking into account that 1023 simulation runs are needed and the all runs are executed with 20 different (sampled) demand data sets, the full enumeration takes many hours to find the optimal solution. As the heuristic is 10 times faster than the full enumeration, a significant reduction in run time is achieved. Further reduction in run time can be achieved by applying a dynamic run length and number of runs, which is beyond the scope of this paper.

## 6. CONCLUSION AND DISCUSSION

In this paper we describe a new method to determine replenishment levels for multiple products simultaneously, where one of the products is a substitute to the others. The method acknowledges the willingness of consumers to buy a substitute product in case of a stock-out. With this strategy, food waste levels for highly perishable products can be reduced. Replenishment decisions for multi-product inventory systems of perishables have not received a lot of attention in literature. Especially the effect of product substitution on these decisions is hardly explored. The problem is practically relevant as there is a strong pressure to reduce food waste, while many retailers struggle in determining the best trade-off between profit, service levels, product waste, and maintaining or even expanding their market share.

The numerical results show one may increase the profit and reduce product waste by exploiting substitution effects when optimising simultaneously the order-up-to levels  $S$ . When the willingness of consumers to substitute is high, it can be beneficial to offer a low product available, i.e. a low service level, for some product, or even to delist it from the assortment. Such a decision depends not only on the willingness to substitute, but also on the (difference in) profit margin, the demand volume, and the shelf life of the multiple products. Though the focus is on profit maximisation, the method is also applied to waste minimisation with a profit constraint to ensure the average profit will not go down. To best of our knowledge no study shows a similar analysis. Furthermore, the method can also be applied to investigate the impact of a service level constraint or a waste constraint on profit and other performance indicators. The trade-off depicted in Figure 1 clearly shows the effect of a high service level on the other parameters.

The results of this study have some similarities with results found in research on lateral transshipments and inventory pooling. With inventory pooling, inventory is shared among different retailers or suppliers [32]. It has proven to reduce the safety inventory, and therefore is often applied in the field of spare part inventory management to reduce backordering and holding costs [31, 32]. In the context of perishable products, a reduction of the safety stock will reduce product waste. However, a main difference between with inventory pooling, is our focus on multiple products and stock-out-based substitution by consumers [33]. Inventory pooling models usually deal with a single product available at different location models and often exclude product perishability.

### 6.1. Managerial insights

The main managerial impact is the quantitative support in determining the service levels that maximise profits, and to determine the price of offering a product at a lower or higher service level. As such the method and

its results support making assortment decisions, as well as pricing decisions (i.e. it provides insights in what the profit margin should be to maximise profit when a desired service level should be met). Similarly, the study shows the profit loss if one aims at a waste level lower than optimal. Additional managerial insights are derived by changing the shelf life, profit margins and demand levels of the products, and by changing consumers willingness to substitute as well as their preference for fresher over older products (FIFO/LIFO). The benefits of combining the products in the replenishment decisions become larger when the shelf life of the products is increased, or when the two products have a different shelf life. However, the benefits are expected to reduce again when the shelf life is increased even more. With a very long shelf life, demand forecasting becomes easier and thus more accurate which will result in higher service levels and lower waste levels without the need of considering the substitution between products. Moreover, when the profit margin of the products differs, anticipating substitution becomes interesting when consumers are willing to substitute towards the product with a higher profit margin. This result is intuitively clear and obviously the impact depends on the degree to which consumers are willing to substitute (when their willingness to substitute is very low the impact is marginal). Consumers in a supermarket will have different purchasing behaviours (i.e. FIFO or LIFO purchase). For all combinations of LIFO and FIFO purchasing behaviour analysed in this paper, the incorporation of the substitution behaviour of consumers shows to influence the retailer performance. In the model, we have assumed a fraction of the excess demand for product 2, is met by product 1, but only after the primary demand for product 1 is met. Hence, we have assumed that substitution does not affect product availability of product 1 to customers with a primary demand for product 1. This assumption is valid as it does not affect the profit and waste levels. It does imply however that the service level  $\beta_{21}$  is underestimated.

### 6.2. Further research

In this research, we presented results for a two-product case, which is common in studies on stock-out based substitution of perishable products. The heuristic is fast and accurate and exploits the structure of the substitution matrix. For all 576 experiments in this study, it does find the optimal replenishment parameters. The heuristic reveals great improvements in terms of numerical efficiency compared to complete enumeration. The set-up of the heuristic facilitates a fast and accurate optimisation. The extension towards two-way substitution is beyond our study and would require a different solution approach. Netessine and Rudi [25] already shows that the inclusion of two-way substitution does not guarantee the profit function to be unimodal in every case. In our study, the profit function shows a clear optimum, due to the

one-way substitution. The heuristic can be extended and applied to more products by extending the search neighbourhood to multiple dimensions. In future research one may investigate such an extension. The current design of the heuristic exploits the structure in the substitution matrix (one-way substitution in this case). For other designs, the running time may increase (more than linearly) in the number of products  $N$ . Besides the extension towards multiple products or different substitution matrices further research could be directed to assortment optimisation the inclusion of other environmental criteria, such as emissions next to product waste, to provide a more holistic view in terms of sustainability.

### ACKNOWLEDGEMENTS

This research is partly funded by the REFRESH project, as part of the Horizon 2020 Framework Programme of the European Union under Grant Agreement no. 641933. The content of this document is the sole responsibility of REFRESH and can in no way be taken to reflect the views of the European Union. This paper has also been supported by The Spanish Ministry (RTI2018-095993-B-100), in part financed by the European Regional Development Fund (ERDF).

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