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Estimating freight rates in inventory replenishment and supplier selection decisions

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Abstract Given the impact of transportation costs on both supplier selection and inventory replenishment decisions in today's enterprises, this article addresses both problems simultaneously by proposing a mixed integer nonlinear programming model to properly allocate order quantities to the selected set of suppliers while taking into account the purchasing, holding and transportation costs under suppliers' capacity and quality constraints. It is shown that incorporating transportation costs in the process of selecting suppliers and establishing an inventory policy, not only affects the order quantity shipped from selected suppliers, but also the actual selection of suppliers. Because of the difficulty that arises when working with actual transportation freight rates in large-scale problems, two continuous functions that estimate actual freight rates are analyzed. It is shown that the use of these functions is very practical and easy to implement.

Keywords Supplier selection · Inventory replenishment · Transportation costs

1 Introduction

This article considers the effect of transportation costs on supplier selection and inventory replenishment decisions. In particular, we study the use of *trucks* as a means of transporting goods and incorporate the transportation cost as a function of the shipment quantity. Companies often need to determine if it is more cost-effective to order smaller shipments from selected suppliers more frequently at a higher per unit shipping cost, or to order larger shipments less frequently, which increases the holding cost at the manufacturing facility. Therefore, to derive inventory policies that simultaneously determine how much, how often and from which suppliers to order, purchasing, holding, and transportation costs are considered.

Despite the importance of transportation costs in supplier selection and order quantity allocation, existing inventory models in the literature have typically assumed that transportation costs are either managed by suppliers, and therefore considered a part of the unit price, or managed by the buyer, and accordingly included as part of the purchasing cost. However, models with such assumptions are insensitive to the effect of the shipment quantity on the per-shipment cost of transportation and seem unrealistic in situations where goods are moved in smaller size, less than truckload (LTL) shipments [23]. One difficulty in trying to incorporate transportation costs into the analysis is the nature of the actual freight rate structure. Trucking companies offer discounts on the freight rate to encourage shippers to buy larger quantities (freight rates take the form of a step function with a decreasing rate as shipping weights increase). Two problems have been recognized when trying to incorporate actual freight rates into inventory models [15]: (1) determining the exact rates between origin and destination is time-consuming and expensive; and (2) the freight rate function is not differentiable.

The remainder of this article is organized as follows. In Sect. 2, previous works related to transportation and inventory management decisions are summarized. In

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Sect. 3, a discussion on actual transportation freight rates is presented. Section 4 provides the description, assumptions and notation of the proposed model. Section 5 introduces two continuous functions that are used to estimate actual freight rates in the proposed model. The development of the proposed model is introduced in Sect. 6. Finally, an illustrative example and some important conclusions derived from this study are provided in Sects. 7 and 8, respectively.

2 Literature review

In this section, different ways in which researchers have incorporated the transportation cost into inventory management decisions are examined. Baumol and Vinod [2] proposed an inventory theoretic model integrating transportation and inventory costs. Their approach incorporates three elements of transportation: cost of shipping (constant shipping cost/unit), speed (mean lead time) and reliability (variance of lead time). Their model assumes a constant unit shipping cost and does not deal with freight rate discounts. Other researchers have used this theoretic model as a foundation for further development. For instance, Das [5] extended Baumol and Vinod's model to allow for independent order quantity and safety stock decisions. Another example is the model proposed by Buffa and Reynolds [3]. Their model includes the rates for LTL, truckload (TL), and carload (CL) shipments. Although the transportation cost was still considered to be constant per unit shipped, they used indifference curves to perform a sensitivity analysis by changing the values of the transportation factors. They concluded that the order quantity was sensitive to the tariff rate, moderately sensitive to the variability in lead time and insensitive to the mean lead time.

Langley [10] used an explicit enumeration procedure to determine the optimal order quantity for a transportation step function (equivalent to freight rate discounts). From this analysis, Carter and Ferrin, Larson and Tyworth [4, 11, 19] continued to use enumeration techniques to determine the optimal order quantity while explicitly considering the actual freight rate structure. Rieksts and Ventura [16] investigated models with TL and LTL transportation costs. They derived optimal policies for both infinite and finite planning horizons that allow a combination of the two transportation modes as an alternative to using a unique option exclusively. The LTL rates are assumed to be constant per unit shipped.

Mendoza and Ventura [13] studied the case of a single manufacturer and multiple suppliers, and proposed a mixed integer nonlinear programming model to optimally allocate order quantities to the selected set of suppliers while taking into account inventory and transportation costs simultaneously. They assumed that goods from suppliers are transported using trucks. Actual transportation costs were modeled with a piecewise linear function using binary variables.

Due to the complexity of the structure of the actual transportation freight rates, the use of freight rate continuous functions to estimate actual freight rates has been repeatedly addressed in the literature, especially to solve large-scale problems. Warsing [23] stated two key advantages for using continuous functions. One is that continuous functions do not require the explicit specification of rate break points for varying shipment sizes nor do they require any embedded analysis to determine if it is economical to increase, over-declare, the shipping weight on a given route. Another advantage is that continuous functions can be used in a wide variety of optimization models.

Swenseth and Godfrey [17] studied five alternative freight rate (continuous) functions: constant, proportional, exponential, inverse and adjusted inverse. They evaluated these functions on their ability to estimate the actual freight rates. Later, Swenseth and Godfrey [18] recommended the use of the inverse and adjusted inverse freight rate functions to approximate actual freight rates while determining the optimal order quantity. In this case, the function that best estimates the TL cost is the inverse function. Conversely, LTL is best estimated by means of the adjusted inverse function. In overcoming some of the lack of fit from the functions proposed by Swenseth and Godfrey, especially in the case of LTL, Tyworth and Ruiz-Torres [20], and Tyworth and Zeng [21] proposed the use of a power function to model LTL freight rates.

To our knowledge, Mendoza and Ventura [13] is the only publication that effectively links the issue of order quantity allocation in the supplier selection problem with multiple suppliers while considering inventory and transportation costs simultaneously. However, their model only works efficiently for small to medium-size problems. Because of the difficulty that arises when working with actual transportation freight rates in large-scale problems, in this article we extend their model by using two existing continuous functions to estimate the actual freight rates.

3 Analysis of actual transportation freight rates

Freight can be transported using TL or LTL. According to Swenseth and Godfrey [18], TL rates are usually stated on a per-mile basis and LTL rates are generally stated per *hundred weight* (CWT) for a given origin and destination. Table 1 shows an example of freight rates for a particular shipping route (extracted for illustrative purposes from [18]).

Table 1 Example of nominal freight rates

Weight range (lb)	Freight rate
Minimum charge	$40.00
1–499	$17.60/CWT
500–999	$14.80/CWT
1,000–1,999	$13.80/CWT
2,000–4,999	$12.80/CWT
5,000–9,999	$12.40/CWT
10,000–19,999	$6.08/CWT
20,000–46,000[a]	$1,110.00

[a] TL capacity

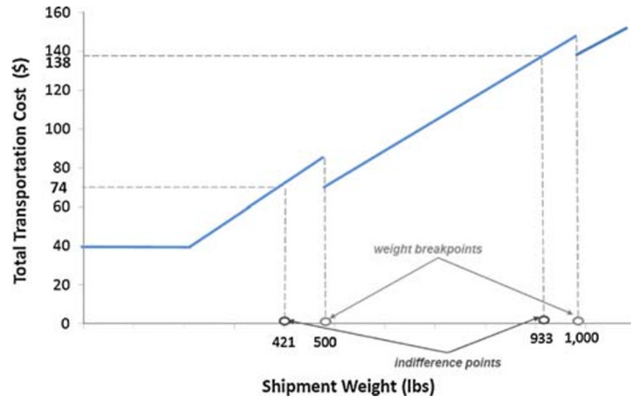


Fig. 2 Total transportation cost structure as typically stated

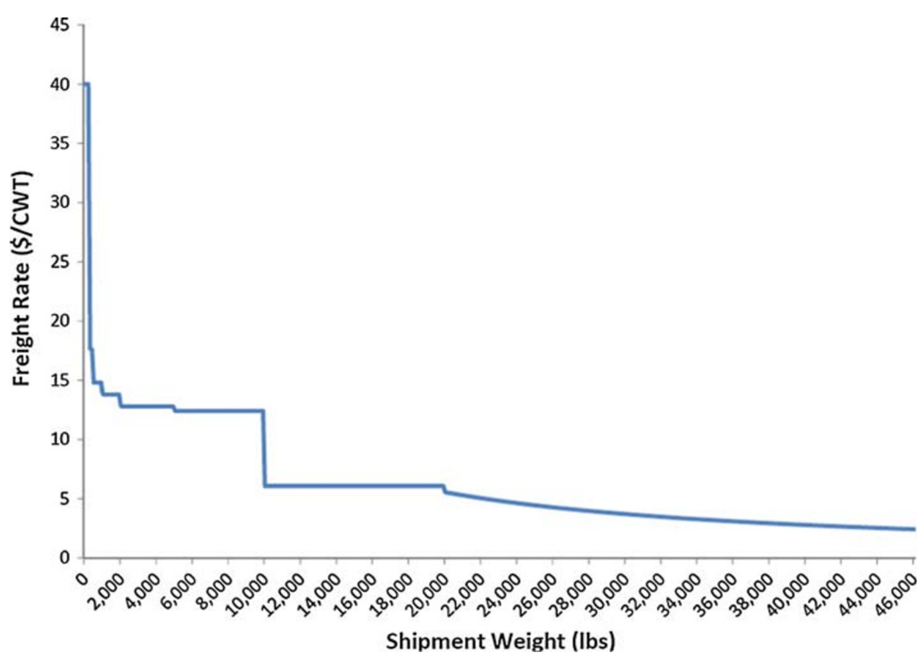
Figure 1 shows a graphical representation of the freight rates versus the weight shipped using the data from Table 1. Notice that freight rates take the form of a step function with a decreasing rate as shipping weights increase. This reflects the economies of scale for larger shipping weights.

Now, Fig. 2 graphically represents the weight shipped (lb) with its corresponding total transportation cost ($) for the rates given in Table 1 (weight is only shown up to 1,010 lb).

Observe from Fig. 2 that there exist some weights that when multiplied by its corresponding freight rate will yield the same total cost as that of the next *weight break point*. These points are called *indifference points* and give rise to the concept of '*over-declare*'. Over-declared shipments are used by shippers to achieve a lower total transportation cost. This is accomplished by artificially inflating the weight to a higher weight break point resulting in a lower total cost [17]. For example, if the weight shipped is between 421 and 500 lb, then the shipment can be over-declared to 500 lb. In this way, the company is charged a fixed amount of $74. The *effective rate* for a given shipment in this range is calculated as the fixed amount of $74 divided by the weight shipped.

By finding all the indifference points from the rates in Table 1, a schedule of actual freight rates can be created. This schedule alternates between ranges of a constant charge per CWT followed by a fixed charge, which is the result of over-declaring a LTL shipment to the next LTL weight break or the TL shipment. In this way, the total transportation cost function shown in Fig. 2 becomes continuous. Although the function shown in Fig. 3 is continuous, it is non-differentiable due to the indifference points and can only be represented as a piecewise linear

Fig. 1 Freight rate versus weight shipped



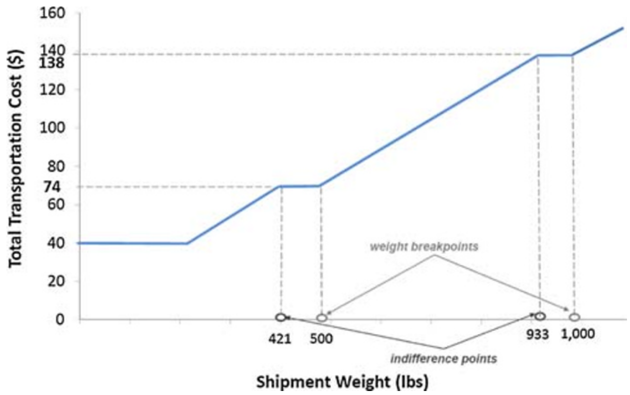


Fig. 3 Total transportation cost function as typically charged

function. Hence, it becomes difficult to incorporate actual freight rates into analytical models.

As addressed in Sect. 2, because of the difficulty that arises when working with actual freight rates, especially for large-scale problems, several researchers have proposed the use of *continuous functions* to properly estimate actual freight rates. These functions are also differentiable and can be represented by a single mathematical expression. In this article, we use two existing continuous functions to estimate the transportation freight rates, which in turn are used to determine the total transportation cost in the proposed supplier selection and order quantity model. These functions are introduced in Sect. 5.

4 Problem description and assumptions

In this research, a single-stage system is studied. The system consists of a manufacturing facility that processes items procured from a set of selected suppliers. Although the manufacturer is assumed to have infinite production capacity, suppliers have finite capacity and provide raw materials (unprocessed items) at different purchasing cost and quality. The problem is determining the order quantity and the number of orders per (repeating) order cycle allocated to each selected supplier, while minimizing the total cost per time unit. The total cost includes purchasing, holding and transportation costs. The inbound transportation cost of the manufacturer is modeled using LTL rates.

In addition to the basic EOQ assumptions [9], such as constant demand rate, no shortages allowed, constant lead times from suppliers, infinite production rate and constant order quantities, it is assumed that the terms with the suppliers are "FOB (free-on-board) origin with freight charges collect", which means that the buyer (manufacturer) pays the freight charges and also owns the goods in transit [8].

The following notation is used throughout this article:

Data

- r Number of available suppliers
- d Demand per time unit
- w Weight of an item
- h Inventory holding cost per item and time unit
- k_i Fixed ordering cost of ith supplier (in \$/order)
- p_i Unit price of ith supplier (in \$)
- c_i Capacity of ith supplier per time unit
- q_i Perfect rate of ith supplier
- q_a Minimum acceptable perfect rate of parts
- l_i Lead time of ith supplier

Variables

- J_i Number of orders of ith supplier per order cycle
- Q Ordered quantity from selected suppliers (in units)
- T Time between consecutive orders
- T_c (repeating) Order cycle time

We assume that production processes from suppliers are imperfect and, therefore, defective parts can be produced. Then, the perfect rate q_i (with values between 0 and 1) represents the proportion of acceptable parts from suppliers that can be used by the manufacturing facility to produce high quality products. Additionally, notice that the orders allocated to suppliers are always of the same size (Q). Having a single Q makes the time between consecutive orders (T) constant, which simplifies the implementation of the inventory policy and helps to coordinate the inventory between consecutive stages when the current facility (e.g., manufacturer) represents only one stage of a multi-stage supply chain network. If the facility under consideration is independent, then it may be better to extend the model to the case where different Q_is are used for the suppliers.

Since the demand rate is constant, the following can be stated: $T = Q/d$. In one order cycle, T_c, there will be $\sum_{i=1}^{r} J_i$ orders placed to the selected suppliers. This implies that multiple orders to one supplier are allowed within an order cycle. After all orders in one order cycle have been placed, the cycle is repeated. For this reason, T_c is defined as '*repeating order cycle time*'. To avoid confusion, from now on this concept is simply referred to as *order cycle*. Thus, the length of an order cycle becomes $T_c = T \cdot \sum_{i=1}^{r} J_i = (Q/d) \cdot \sum_{i=1}^{r} J_i$. Hence, the total number of order cycles per time unit is given by $1/T_c = 1/(T \cdot \sum_{i=1}^{r} J_i) = d/(Q \cdot \sum_{i=1}^{r} J_i)$.

5 Estimating freight rates with continuous functions

In this section, two continuous functions used to fit the actual freight rates in this article are introduced. As mentioned in Sect. 2, Swenseth and Godfrey [17] proposed the use of the proportional function to model LTL freight rates. The function is as follows:

$$F_y = F_x + \alpha(W_x - W_y), \tag{1}$$

where F_y is the freight rate for shipping a given load ($/CWT), F_x is the TL rate per CWT, W_x is the TL weight (lb), W_y is the shipping weight (lb), and α represents the rate at which the freight rate increases per 100 lb decrease in shipping weight. Notice that the terms F_x and αW_x are constants and can be substituted by another constant (say, A), and that W_y can also be expressed as $W_y = Qw$, where Q is the order quantity and w is the weight of the item under consideration. Hence, Eq. 1 can be rewritten as

$$F_y = A - \alpha Qw. \tag{2}$$

Equation 2 is the proportional function proposed by Langley [10]. It is easy to see that the freight rate decreases at a rate αw for every unit increase in Q. The value of α can be obtained in two ways [15]: (1) from Eq. 1 by minimizing the mean squared error between actual and estimated LTL freight rates for each route. In this case, rates are generated over a realistic range of shipment quantities (Q) and then a curve is fitted to the rate data; (2) from Eq. 2 by fitting a simple linear regression model between the freight rate and order quantity.

The second way to obtain α is typically used when the transportation freight rates are known (e.g., nominal freight rates are given for a specific route). DiFillipo [6] and Natarajan [15] report the use of Langley's proportional function in actual implementations. In this article, we assume that the freight rates from potential suppliers are known. Therefore, we fit a simple linear regression model between the freight rates and the corresponding weight range to find the value of α (and simultaneously the value of the constant A). In this way, we obtain the function that estimates the freight rates for shipping a given load from a particular supplier (in the form of Langley's proportional function).

In addition to Langley's function, a method to generate a power function has been proposed by Tyworth and Ruiz-Torres [20]. The general form of this estimate as a function of the weight shipped is as follows:

$$F_y = a(Qw)^b, \tag{3}$$

where a and b are the corresponding coefficients. These coefficients can be found using nonlinear regression analysis. However, notice that Eq. 3 can also be expressed as follows:

$$\ln(F_y) = \ln[a(Qw)^b] = \ln(a) + b\ln(Qw). \tag{4}$$

In this way, the coefficients can also be found by performing a simple linear regression analysis.

To generate the rate functions, *effective rates* need to be computed, as explained in Sect. 3. Figure 4 shows the

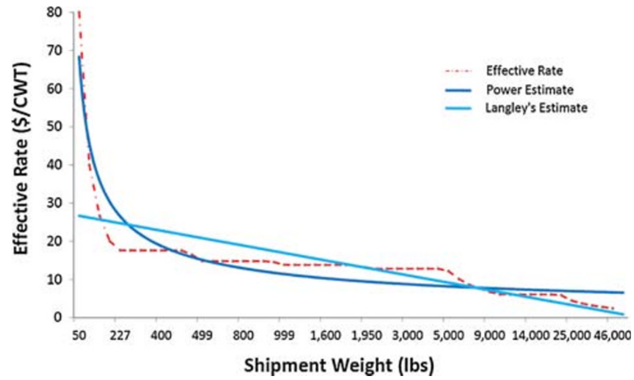


Fig. 4 Langley's and power function estimates

continuous functions generated using Eqs. 2 and 3 to fit the freight rates for the data in Table 1.

6 Inventory replenishment and supplier selection model

6.1 Estimating transportation costs for the proposed model

Let F_{y_i}, $i = 1,\ldots,r$, be the freight rate function for shipping a given load from supplier i ($/CWT). Thus, the transportation freight rate from supplier 'i' using *Langley's proportional function* (Eq. 2) is

$$F_{y_i} = A_i - \alpha_i Qw, \tag{5}$$

and the transportation cost for a shipment quantity Q from supplier 'i' using Eq. 5 is

$$(A_i + \alpha_i Qw)\frac{Qw}{100}. \tag{6}$$

Since freight rates are given in $/CWT, the order quantity to be shipped is multiplied by the weight of the item (w) and divided by 100 to express the weight shipped in CWT. The transportation cost per time unit is obtained by multiplying expression (6) by the total number of orders allocated to all suppliers in one order cycle $\left(\sum_{i=1}^{r} J_i\right)$ and by the total number of order cycles per time unit $\left(d/(Q \cdot \sum_{i=1}^{r} J_i)\right)$,

$$\sum_{i=1}^{r}\left\{(A_i + \alpha_i Qw)\frac{J_i Qw}{100}\cdot\frac{d}{Q}\cdot\frac{1}{\sum_{i=1}^{r} J_i}\right\} = \frac{dw}{100}\sum_{i=1}^{r}\left\{(A_i + \alpha_i Qw)\frac{J_i}{\sum_{i=1}^{r} J_i}\right\}. \tag{7}$$

Similarly, the transportation freight rate from supplier 'i' using the *power function* (Eq. 3) is

$$F_{y_i} = a_i(Qw)^{b_i}, \tag{8}$$

and the total transportation cost for shipping an order quantity (Q) from supplier 'i' using Eq. 8 is

$$\left\{a_i(Qw)^{b_i}\right\}\frac{Qw}{100}. \quad (9)$$

Finally, its corresponding transportation cost per time unit is expressed as follows:

$$\frac{dw}{100}\sum_{i=1}^{r}\left\{\left(a_i(Qw)^{b_i}\right)\frac{J_i}{\sum_{i=1}^{r}J_i}\right\}. \quad (10)$$

6.2 In-transit inventory

Since FOB origin, freight collect, is assumed, the manufacturer not only pays for freight charges but is also responsible for goods in transit. Therefore, the in-transit inventory should also be reflected in the total inventory per time unit held by the manufacturer. The in-transit inventory cost per time unit for each supplier is

$$\frac{l_i}{Y}\cdot\frac{dJ_i}{\sum_{i=1}^{r}J_i}\cdot h, \quad (11)$$

where Y is the number of days per time unit. The first term (l_i/Y) represents the fraction of time that an order of size Q spends in transit. The second term represents the fraction of the total demand procured from supplier i, and h is the holding cost rate (herein assumed to be the same as the regular holding inventory cost). The final expression for in-transit inventory per time unit considering all suppliers is

$$\frac{dh}{Y}\cdot\frac{\sum_{i=1}^{r}l_iJ_i}{\sum_{i=1}^{r}J_i}, \quad (12)$$

where $\sum_{i=1}^{r}l_iJ_i/\sum_{i=1}^{r}J_i$ indicates an average lead time. Note that the in-transit inventory cost does not depend on the order quantity Q. While this cost does not directly affect the size of the order, it does affect the number of orders allocated to suppliers (J_i's).

6.3 Proposed model with continuous functions

The total cost per time unit considering continuous functions to estimate the transportation freight rates is the following:

$$Z_{FQ} = \frac{d}{Q}\cdot\frac{\sum_{i=1}^{r}J_ik_i}{\sum_{i=1}^{r}J_i} + d\cdot\frac{\sum_{i=1}^{r}J_ip_i}{\sum_{i=1}^{r}J_i} + \frac{hQ}{2} + \frac{dh}{Y}\cdot\frac{\sum_{i=1}^{r}J_il_i}{\sum_{i=1}^{r}J_i} + \frac{dw}{100}\cdot\frac{\sum_{i=1}^{r}J_i\cdot F_{y_i}}{\sum_{i=1}^{r}J_i}. \quad (13)$$

Recall that we consider three types of costs: purchasing (or ordering), holding and transportation. The purchasing cost has two components: (1) fixed ordering cost (first term), which is obtained by dividing the total ordering cost per order cycle, $\sum_{i=1}^{r}J_ik_i$, by the length of the order cycle, $T_c=(Q/d)\cdot\sum_{i=1}^{r}J_i$; and (2) variable ordering cost (second term), where $\sum_{i=1}^{r}J_ip_i/\sum_{i=1}^{r}J_i$ indicates an average price of a purchased unit. We also have two components of the holding cost: (1) holding cost at the manufacturing facility (third term); since the order quantity (Q) is the same for all suppliers, the holding cost per time unit is simply expressed as the unit holding cost times the average inventory on hand, $Q/2$; and (2) in-transit inventory (fourth term), as per expression (12) in Sect. 6.2. Finally, the fifth term accounts for the transportation cost. In this case, Eq. 5 replaces F_{y_i} when Langley's (linear) function is used to estimate the actual freight rate from supplier i. Likewise, Eq. 8 replaces F_{y_i} when the power function is employed to estimate the freight rate from supplier i. This is equivalent to replacing the fifth term in Eq. 13 with expressions 7 or 10.

Two types of constraints are considered in the model: capacity and quality. The capacity constraints are as follows:

$$d\cdot\frac{J_j}{\sum_{i=1}^{r}J_i}\le c_j, \quad \text{for} \quad j=1,\ldots,r, \quad (14)$$

where the left-hand side represents the proportion of demand per time unit that is assigned to the ith supplier, which is limited by its offered capacity per time unit (right-hand side). The quality constraint is

$$\frac{\sum_{i=1}^{r}J_iq_i}{\sum_{i=1}^{r}J_i}\ge q_a, \quad (15)$$

where the left-hand side represents the average perfect rate offered by suppliers. This average needs to meet the minimum acceptable perfect rate (q_a) imposed by the manufacturer. This manufacturer's perfect rate represents an average quality level that needs to be maintained. Notice that, if this perfect rate were a lower bound for all suppliers, the suppliers with a lower perfect rate would have to be rejected and the constraint would be unnecessary.

By including capacity and quality constraints, and rearranging terms of Eq. 13, the complete formulation for the supplier selection and order quantity allocation problem considering transportation costs is the following:

$$\begin{aligned}
\text{(P1)}\quad \text{minimize}\quad & Z_{FQ}=\frac{d}{M}\left[\frac{1}{Q}\cdot\sum_{i=1}^{r}J_ik_i+\sum_{i=1}^{r}J_ip_i + \frac{w}{100}\cdot\sum_{i=1}^{r}J_iF_{y_i}+\frac{h}{Y}\cdot\sum_{i=1}^{r}J_il_i\right]+\frac{hQ}{2},\\
\text{subject to}\quad & dJ_i\le c_iM, \quad i=1,\ldots,r,\\
& \sum_{i=1}^{r}J_iq_i\ge q_aM,\\
& \sum_{i=1}^{r}J_i=M,\\
& Q\ge 0,\\
& J_i\ge 0, \quad \text{integer}, \quad i=1,\ldots,r,\\
& M\ge 1, \quad \text{integer}.
\end{aligned}$$

Notice that the total number of orders allocated to all selected suppliers in one order cycle, $\sum_{i=1}^{r}J_i$, has been defined as M. The reason is that if the optimal value of M that minimizes the total cost per time unit results in an excessively large order cycle time, then the manufacturer may restrict M to a reasonably small value to reduce the entire order cycle. Short cycle times facilitate interaction with suppliers and simplify the inventory replenishment process.

7 Illustrative example

In this section, a numerical example is presented to analyze the impact of transportation costs on supplier selection and order quantity allocation decisions. To highlight the advantages of using Langley's (linear) and the power functions to estimate the actual transportation freight rates, it is important to compare the solutions obtained from these estimates to the absolute optimal solution obtained by Mendoza and Ventura [13]. Important properties and conclusions are derived from this analysis.

7.1 Input data

The example problem consists of one manufacturer and three potential suppliers. The manufacturer needs to decide its inventory policy with respect to a component part needed in the assembly process. The weight of the component part is $w = 16$ lb and its demand has been estimated at $d = 1{,}000$ units/month with a corresponding holding cost of $h = \$10$/units per month. The manufacturer has specified its minimum acceptable perfect rate as $q_a = 0.95$. Table 2 shows additional data for the three potential suppliers.

The suppliers are located in different geographical areas, and therefore, the corresponding freight rates are different. The capacity of the trucks is $W_x = 40{,}000$ lb. Tables 3, 4 and 5, respectively, show the nominal [1] and actual freight rates for suppliers 1, 2 and 3. The *actual and effective freight rates* were calculated as explained in Sect. 3. Additionally, their corresponding TL rates are: \$18.8125/CWT (\$7,525/TL), \$33/CWT (\$13,200/TL) and \$12.575/CWT (\$5,030/TL).

The functions generated by fitting Eqs. 5 and 8 to the *effective rates* of each supplier are summarized in Table 6 along with their corresponding coefficient of determination (R^2). The coefficient of determination is widely used to determine how well a regression function fits the data [22]. In this particular case, R^2 is a measure of how well the estimates shown in Table 6 fit the actual freight rates. The analysis was performed using [14]. As shown in Table 6, all R^2 values are greater than 0.93. Thus, all three power functions provide very good estimates of the actual freight rates.

Table 2 Data for suppliers

Supplier I	Price (p_i) (\$)	Fixed ordering cost (k_i) (\$)	Perfect rate (q_i)	Capacity (c_i) (units/month)	Leadtime (l_i) (days)
1	20	160	0.93	700	1
2	24	140	0.95	800	3
3	30	130	0.98	750	2

Table 3 Nominal and actual freight rates for supplier 1

Nominal freight rate		Actual freight rate	
Weight range (lb)	Freight rate	Weight range (lb)	Freight rate
1–499	\$107.75/CWT	1–428	\$107.75/CWT
500–999	\$92.26/CWT	429–499	\$461.3
1,000–1,999	\$71.14/CWT	500–771	\$92.26/CWT
2,000–4,999	\$64.14/CWT	772–999	\$711.4
5,000–9,999	\$52.21/CWT	1,000–1,803	\$71.14/CWT
10,000–19,999	\$40.11/CWT	1,804–1,999	\$1,282.8
20,000–29,999	\$27.48/CWT	2,000–4,070	\$64.14/CWT
30,000–40,000	\$7,525	4,071–4,999	\$2,610.5
		5,000–7,682	\$52.21/CWT
		7,683–9,999	\$4,011
		10,000–13,702	\$40.11/CWT
		13,703–19,999	\$5,496
		20,000–27,383	\$27.48/CWT
		27,384–40,000	\$7,525

Table 4 Nominal and actual freight rates for supplier 2

Nominal freight rate		Actual freight rate	
Weight range (lb)	Freight rate	Weight range (lb)	Freight rate
1–499	\$136.26/CWT	1–403	\$136.26/CWT
500–999	\$109.87/CWT	404–499	\$549.35
1,000–1,999	\$91.61/CWT	500–833	\$109.87/CWT
2,000–4,999	\$79.45/CWT	834–999	\$916.1
5,000–9,999	\$69.91/CWT	1,000–1,734	\$91.61/CWT
10,000–19,999	\$54.61/CWT	1,735–1,999	\$1,589
20,000–29,999	\$48.12/CWT	2,000–4,399	\$79.45/CWT
30,000–40,000	\$13,200	4,400–4,999	\$3,495.5
		5,000–7,811	\$69.91/CWT
		7,812–9,999	\$5,461
		10,000–17,623	\$54.61/CWT
		17,624–19,999	\$9,624
		20,000–27,431	\$48.12/CWT
		27,432–40,000	\$13,200

Table 5 Nominal and actual freight rates for supplier 3

Nominal freight rate		Actual freight rate	
Weight range (lb)	Freight rate	Weight range (lb)	Freight rate
1–499	\$81.96/CWT	1–428	\$81.96/CWT
500–999	\$74.94/CWT	429–499	\$374.7
1,000–1,999	\$61.14/CWT	500–771	\$74.94/CWT
2,000–4,999	\$49.65/CWT	772–999	\$611.4
5,000–9,999	\$39.73/CWT	1,000–1,803	\$61.14/CWT
10,000–19,999	\$33.44/CWT	1,804–1,999	\$993
20,000–29,999	\$18.36/CWT	2,000–4,070	\$49.65/CWT
30,000–40,000	\$5,030	4,071–4,999	\$1,986.5
		5,000–7,682	\$39.73/CWT
		7,683–9,999	\$3,344
		10,000–13,702	\$33.44/CWT
		13,703–19,999	\$3,672
		20,000–27,383	\$18.36/CWT
		27,384–40,000	\$5,030

7.2 Analysis of results

The following results are labeled for the purpose of simplifying the analysis:

1. *Solution to the problem without transportation + actual transportation cost* (*WTA*): results are obtained in three steps. First, problem (P1) is solved ignoring the transportation and in-transit inventory terms. Here, the idea is to find the total cost per time unit (Z_S^*), the order allocation (J_i's) and the order quantity allocated to selected suppliers (Q^*). Second, the transportation and in-transit inventory costs are computed separately, as explained. The actual freight rates for the three suppliers are obtained from Tables 3, 4, 5 considering the shipping weight $Q^* \cdot w$. Then, the transportation cost per time unit is given by

$$\frac{dw}{100} \cdot \frac{\sum_{i=1}^{r} J_i F_i^A}{\sum_{i=1}^{r} J_i}, \tag{16}$$

where F_i^A indicates the actual freight rates obtained from tables (\$/CWT). The in-transit transportation cost per time unit is calculated as follows:

$$\frac{dh}{Y} \cdot \frac{\sum_{i=1}^{r} J_i l_i}{\sum_{i=1}^{r} J_i}. \tag{17}$$

Third, the resulting costs from Eqs. 16 and 17 are added to the cost found in step one, Z_S^*.

2. *Langley's function* (*LF*): results are obtained by solving problem (P1) using the Langley's freight rate functions (F_{y_1}, F_{y_2}, and F_{y_3}) provided in the second column of Table 6. These results consider *estimated* transportation and inventory costs simultaneously.
3. *LF with actual transportation costs* (*LFA*): these results are calculated in two steps. First, using the order quantity obtained in LF, the corresponding actual freight rates for the selected suppliers are determined from Tables 3, 4, 5. Second, the total transportation cost is recalculated using these actual freight rates in Eq. 16.
4. *Power function* (*PF*): results are obtained by solving problem (P1). The freight rate (power) functions (F_{y_1}, F_{y_2}, and F_{y_3}) are provided in the fourth column of Table 6. These results consider *estimated* transportation and inventory costs simultaneously.
5. *PF with actual transportation costs* (*PFA*): these results are found in two steps. First, using the order quantity from PF, the corresponding actual freight rates for the selected suppliers are determined from Tables 3, 4, 5. Second, the total transportation cost is recalculated using these actual freight rates in Eq. 16.
6. *Absolute optimal solution* (*AO*): results are obtained by solving the mathematical model proposed by Mendoza and Ventura [13], see Appendix 1. Their model provides the absolute optimal by representing transportation costs as a continuous piecewise linear function (of the weight shipped) using binary variables.

LFA and PFA are calculated to compare the actual costs with the cost of the absolute optimal (AO) solution. Table 7 shows the solution of the illustrative example for cases 1–6. Notice that the order quantity of WTA is smaller than those obtained for LFA, PFA and AO. The reason is that by incorporating transportation and inventory costs simultaneously, as in LFA, PFA and AO, the manufacturer can take advantage of economies of scale in shipping. After adding the transportation and in-transit inventory costs to

Table 6 Summary of freight rate continuous function estimates

Supplier 1	Langley's Fn (\$/CWT)	R^2 value	Power Fn (\$/CWT)	R^2 value
1	$F_{y_1} = 61.7 - 0.00127\,(Qw)$	0.763	$F_{y_1} = 1586.21\,(Qw)^{-0.4028}$	0.947
2	$F_{y_2} = 80.3 - 0.00129\,(Qw)$	0.746	$F_{y_2} = 789.97\,(Qw)^{-0.2831}$	0.935
3	$F_{y_3} = 48.2 - 0.00109\,(Qw)$	0.758	$F_{y_3} = 2247.57\,(Qw)^{-0.4757}$	0.938

Table 7 Solutions for the illustrative example (same Q for selected supplier)

	Order allocation			Order quantity	Cycle's length	Total cost	% Deviation
	J_1	J_2	J_3	Q	T_c (month)	($/month)	(from AO)
WTA	3	20	2	168	4.2	38,346.10	13.4
LF[a]	3	0	2	277	1.4	34,544.44	2.14
LFA	3	0	2	277	1.4	34,917.50	3.40
PF[a]	3	0	2	551	2.8	33,322.39	−1.47
PFA	3	0	2	551	2.8	34,283.30	1.40
AO	3	0	2	625	3.1	33,819.10	–

[a] These cases consider *estimated* transportation costs

Table 8 Solution for the illustrative example (different Q_is for the selected suppliers)

	Order allocation			Order quantities			Total cost
	J_1	J_2	J_3	Q_1	Q_2	Q_3	($/month)
AO (Q_i's)	3	0	4	625	0	313	33,679.95

WTA, the worst total cost per time unit is achieved (13.4% greater than that of AO). Moreover, the solutions in LFA, PFA and AO obtain a different allocation of orders to suppliers compared to that of WTA. These solutions eliminate supplier 2 altogether, mainly due to its high average actual freight rates. Observe also that the order allocations for LFA, PFA and AO are the same. Hence, by solving problem (P1) with the freight rate continuous functions, one can obtain the optimal number of orders allocated to each selected supplier.

Now, for the same input data used in this section, Table 8 shows the computational results of the optimal solution obtained by solving the mathematical model proposed by Mendoza and Ventura [13] for the case in which different Q_is are allowed for each selected supplier. This solution includes the values of each Q_i and the objective function. Notice that the improvement in the objective function, compared to the solution in which similar Qs are considered for each selected supplier, is insignificant (less than 1%).

In terms of the complexity in solving the proposed model versus the AO model proposed by Mendoza and Ventura [13] for the numerical problem with similar Qs discussed so far, Table 9 indicates some advantages for using the proposed model (with approximations). Observe that the number of linearizing constraints decreases substantially from Mendoza and Ventura's model to the proposed model. Additionally, the difference in runtime between the two models is over 25 s. Clearly, in large-scale real-world applications, the number of linearizing constraints, binary variables and runtime will increase substantially when using the AO model. Hence, the proposed model is much easier to implement and use in practice.

Table 9 Comparison between Mendoza and Ventura's model and the proposed model

Type of constraints	AO[a] [13]	Proposed model (LFA, PFA)[b]
Capacity	3	3
Quality	1	1
Number of orders allocated (M)	1	1
Linearizing constraints	57	0
Binary variables	42	0
Integer variables	3	3
Total	107	8

[a] Runtime = 26 s

[b] Runtime <1 s

Figure 5 shows the steps required to implement each model.

Observe that the proposed model can entirely be implemented using commercial software available in spreadsheets such as Microsoft EXCEL; whereas the model proposed by Mendoza and Ventura [13], because of the added complexity in terms of number of linearizing constraints and binary variables, requires specialized optimization software, such as LINGO [12] or GAMS [7], which is more expensive.

To study the performance of Langley's (linear) and power functions, an analysis of transportation costs for different values of M was carried out. Table 10 shows the transportation costs of WTA, LFA, PFA and AO for values of M from 2 to 25.

The impact of not considering inventory and transportation costs simultaneously results in an average deviation of 87% from the optimal solution (AO). Essentially, this translates to higher shipping costs. Modeling of freight rates using Langley's (linear) function results in transportation costs that are 43% higher than those of AO. In contrast, estimating the transportation costs with the power function results in a 14% deviation from AO. Although the simple straight line function outperforms WTA, this approximation may not be suitable in practice. However, the power function works very well.

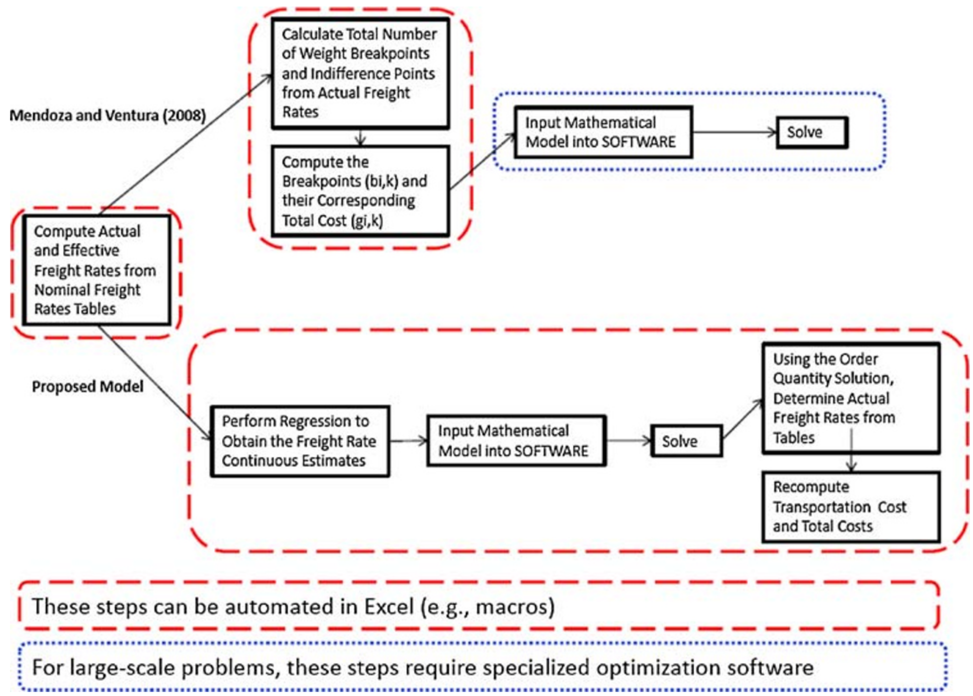


Fig. 5 Implementation steps for both the Mendoza and Ventura's model and the proposed model

Table 10 Comparative analysis of transportation costs

M	WTA	Dev from AO (%)	LFA	Dev from AO (%)	PFA	Dev from AO (%)	AO
2	$9,103.20	55	$8,491.91	44	$6,739.13	15	$5,884.00
3	$10,306.13	51	$9,783.22	43	$7,654.38	12	$6,835.20
4	$10,907.60	85	$8,491.91	44	$6,739.13	15	$5,884.00
5	$9,335.04	56	$8,532.50	42	$6,791.88	13	$5,990.72
6	$11,509.07	96	$9,163.24	56	$7,241.78	23	$5,884.00
7	$11,680.91	96	$8,521.56	43	$6,776.94	14	$5,960.23
8	$10,601.40	68	$9,000.29	43	$7,118.40	13	$6,307.40
9	$10,835.91	82	$8,515.26	43	$6,768.60	14	$5,943.29
10	$11,023.52	84	$8,532.50	42	$6,791.88	13	$5,990.72
11	$11,177.02	79	$8,877.66	42	$7,038.59	13	$6,240.44
12	$11,304.93	89	$8,526.18	43	$6,783.18	14	$5,972.93
13	$11,413.17	85	$8,820.20	43	$6,993.25	13	$6,185.60
14	$11,505.94	93	$8,521.56	43	$6,776.94	14	$5,960.23
15	$11,586.35	93	$8,532.50	42	$6,791.88	13	$5,990.72
16	$11,656.70	89	$8,770.08	42	$6,961.83	13	$6,162.40
17	$11,718.78	96	$8,528.06	43	$6,785.74	14	$5,978.16
18	$11,773.96	92	$8,740.23	43	$6,937.45	13	$6,131.47
19	$11,823.33	98	$8,524.49	43	$6,780.88	14	$5,968.25
20	$11,867.76	98	$8,532.50	42	$6,791.88	13	$5,990.72
21	$11,907.96	95	$8,713.63	42	$6,921.50	13	$6,121.52
22	$11,944.51	100	$8,529.08	43	$6,787.14	13	$5,981.02
23	$11,977.88	96	$8,695.05	43	$6,905.87	13	$6,100.87
24	$12,008.47	101	$8,526.18	43	$6,783.18	14	$5,972.93
25	$12,036.61	101	$8,532.50	42	$6,791.88	13	$5,990.72
Average % deviation		87		43		14	

When solving problem (P1), we assume all shipments from suppliers are LTL. Then, the proposed continuous functions are used to estimate the actual freight rates. If after solving problem (P1), the shipping weight is such that it can be over-declared as a full TL, this means that the continuous functions used to estimate the actual freight rates might be overestimating the transportation cost per time unit. This situation can be corrected by recalculating the transportation cost per time unit with the [lower] freight rate of a full TL. This results in a lower total transportation cost and the order quantity remains unchanged.

8 Conclusions

In this article, the impact of transportation costs in both supplier selection and inventory replenishment decisions has been addressed. Under the assumptions that shipments from suppliers are LTL and order quantities from the selected suppliers are of the same size, two existing continuous functions have been used to estimate the actual freight rates of supplier shipments. The use of these functions is recommended when the number of potential suppliers is large or when no specialized optimization software, such as LINGO or GAMS, is available to solve the problem optimally. These functions do not require specification of rate break points or any embedded analysis to determine when to over-declare a given shipment. Further, fitting continuous functions and solving problem (P1) can easily be done using Microsoft EXCEL. Therefore, the use of continuous functions, especially the power function, to estimate the actual transportation costs makes the model proposed in this article very practical.

Finally, it has been shown that incorporating transportation costs into inventory replenishment decisions, not only affects the order quantity shipped from selected suppliers, but also the actual selection of suppliers. This may actually affect the configuration of supply chains.

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Appendix 1: Mendoza and Ventura's model [13]

The mathematical model for supplier selection and order quantity allocation using actual transportation freight rates from Mendoza and Ventura [13] is presented in this appendix. With the exception of $\lambda_{i,k}$, $b_{i,k}$, $g_{i,k}$, which are introduced and explained below, the notation is the same as that used in the present article:

$$\text{(AO) minimize} \quad Z_A = \frac{d}{M}\left[\frac{1}{Q}\sum_{i=1}^{r} J_i k_i + \sum_{i=1}^{r} J_i p_i + \frac{1}{Q}\sum_{i=1}^{r} J_i \cdot TC_i(Qw) + \frac{h}{Y}\cdot\sum_{i=1}^{r} J_i l_i\right] + \frac{hQ}{2}$$

$$\text{subject to} \quad J_i d \le c_i M, \quad i = 1,\ldots,r, \tag{18}$$

$$\sum_{i=1}^{r} J_i q_i \ge q_a M, \tag{19}$$

$$\sum_{i=1}^{r} J_i = M, \tag{20}$$

$$Q\cdot w = \sum_{k=1}^{u_i+1} b_{i,k}\cdot\lambda_{i,k}, \quad i = 1,\ldots,r, \tag{21}$$

$$TC_i(Qw) = \sum_{k=1}^{u_i+1} g_{i,k}\cdot\lambda_{i,k}, \quad i = 1,\ldots,r, \tag{22}$$

$$\lambda_{i,k} \le Z_{i,k}, \quad i = 1,\ldots,r; \quad k = 1, \tag{23}$$

$$\lambda_{i,k} \le Z_{i,k-1} + Z_{i,k}, \quad i = 1,\ldots,r; \quad k = 2,\ldots,u_i, \tag{24}$$

$$\lambda_{i,k} \le Z_{i,k-1}, \quad i = 1,\ldots,r; \quad k = u_i + 1, \tag{25}$$

$$\sum_{k=1}^{u_i+1} \lambda_{i,k} = 1, \quad i = 1,\ldots,r, \tag{26}$$

$$\sum_{k=1}^{u_i+1} Z_{i,k} = 1, \quad i = 1,\ldots,r, \tag{27}$$

$$Z_{i,k} \in \{0,1\}, \quad i = 1,\ldots,r; \quad k = 1,\ldots,u_i, \tag{28}$$

$$\lambda_{i,k} \ge 0, \quad i = 1,\ldots,r; \quad k = 1,\ldots,u_i + 1, \tag{29}$$

$$Q \ge 0, \tag{30}$$

$$J_i \ge 0, \quad \text{integer}, \quad i = 1,\ldots,r, \tag{31}$$

$$M \ge 1, \quad \text{integer}. \tag{32}$$

The total weight shipped from supplier 'i' (in one order) is defined in Eq. 21, where $b_{i,k}$ (i=1,…,r; k =1,…,u_i+1) represents a break point (lb) that can be obtained from the actual LTL rate structure, and u_i is the total number of break points in the actual LTL rate structure. Furthermore, $b_{i,1} = 0$ and b_{i,u_i+1} equals the capacity of a TL. The total transportation cost charged to supplier i for the weight (Qw) shipped is defined in Eq. 22, where $g_{i,k}$ (i = 1,…,r; k =1,…,u_i+1) is the total transportation cost obtained by evaluating the corresponding break point $b_{i,k}$ into the actual LTL rate structure. As per the definition of b_{i,u_i+1}, g_{i,u_i+1} is the cost of a full TL. Moreover, each binary variable, $Z_{i,k}$, represents one linear segment of the freight rate function (see Fig. 3). By constraint 27, only one $Z_{i,k}$ per supplier can get a value of '1'. Then, the specific segment chosen contains the weight shipped (Qw) and its corresponding total transportation cost $TC_i(Qw)$ is expressed as the linear combination of $\lambda_{i,k}$ and $\lambda_{i,k+1}$ ($0 \leq \lambda_{i,k} \leq 1$, $i = 1,\ldots,r$; $k = 1,\ldots,u_i + 1$). This clearly explains the presence of constraints 23–27.

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