Logistics Research (2021) 14:5
DOI_10.23773/2021_5

Travel time model for multi-deep automated storage and retrieval system with a homogeneous allocation structure

Timo Lehmann · Jakob Hußmann

Received: 5 December 2020 / Accepted: 29 September 2021 / Published online: 15 October 2021

ABSTRACT

This paper presents a travel time model for multi-deep AS/RS, which determines the average travel times during a single command storage or retrieval and a dual command cycle. The model can determine the relocation probability, the number of expected relocations and the travel times in storage channels itself exactly depending on the stock filling level. The travel time model assumes random storage policies. Travel time determination is trivial for single-deep AS/RS, because no relocation necessity applies and it gets more complicated with an increasing depth of the storage racks. The deeper goods can be stored, the more goods can potentially be stored in front of each other in one storage channel. This leads to relocation operations of blocking goods and causes higher total travel times. The higher the stock filling level, the higher is the relocation probability and the number of necessary relocations. The calculation of relocation probabilities in this work is based on a homogeneous storage good allocation structure which leads to a symmetric allocation of storage goods and enables an easy modelling of travel times. This paper presents a travel time model with a continuous storage rack approximation of a multi-deep AS/RS in closed-form expression. Furthermore, the storage channel allocation probabilities are mathematically proven. The relocation probability for storage operations and retrieval operations are the same. Finally, the derived travel time models and relocation probabilities are verified by simulation.

KEYWORDS: travel time models · multi-deep · n-deep · k-deep · automatic storage and retrieval systems · analytic modelling · automated warehouses

Timo Lehmann, Corresponding Author

Jakob Hußmann

Karlsruhe Institute of Technology, GERMANY

1. INTRODUCTION

Automatic storage and retrieval systems (AS/RS) are state of the art since the 1960s and are further developed ever since. To determine the performance of storage and retrieval systems of all sorts, it is necessary to determine travel times of single and dual command cycles. Starting with Gudehus [1], travel time models for AS/RS were introduced and further developed regarding the number of aisles, the number of storage and retrieval (S/R) machines, different I/O-points, dwell points and double-deep racks.

Double-deep AS/RS have the advantage, that more storage goods can be stored in the same area compared to a single-deep AS/RS. This advantage was used in the last years by AS/RS manufacturers, which developed systems that are triple- or even quadruple-deep. The improved usage of space causes the need of relocations when a storage good is blocked by other storage goods, which leads to higher travel times. In triple- or quadruple-deep AS/RS, relocations of up to two or three blocking storage goods may be necessary. In the future AS/RS with even larger depth seem to be realistic. Currently, the academic research of travel time models lags behind these innovations of AS/RS manufacturers and does not fully cover the relevance of relocations.

In recent years, several variations of such multi-deep AS/RS were introduced, such as the vehicle AS/RS [2], flow-rack AS/RS [3] or compact AS/RS with powered or gravity conveyors inside the rack itself [4]. These variations have a different functionality compared to the most basic version of an AS/RS, which has one

S/R machine in each aisle, which cannot leave this aisle and which can transport only one storage unit at a time [2]; therefore also the travel times are different. Nevertheless, some aspects can be compared such as the storage good allocation structure and the relocation probabilities.

The academic research of travel time models for multi-deep AS/RS can be improved in three aspects: Firstly, many travel time models avoid the necessity of relocations through either a limitation to batches [5], a nonregarding of relocations [6] or automatic relocation in the racks itself [7]. Secondly, it is assumed, that the AS/RS operates always with a very high stock filling level, so that in a case of relocation, the maximal number of relocations has to be performed [8][9]. Thirdly, random storage and relocation strategies are assumed but not consistently implemented. For example, empty storage channels are excluded at high stock filling levels [3], although it has been shown that these exist even at high stock filling levels, when random storage and relocation strategies are applied [10].

This paper introduces a new storage channel allocation – inspired by Lippolt [10] – which enables the calculation of travel times in multi-deep AS/RS depending on the rack depth, the stock filling level and a possibility of exact determination of relocations probabilities with a continuous storage rack approximation. All storage operations choose a random storage location and retrieval operations choose randomly from all stored goods. Possible relocations are executed in a way which preserves the current AS/RS allocation structure except for the affected storage location. This is achieved using predetermined relocation operations and ensures the *homoge neous storage channel allocation*, which allows an exact calculation of relocation probabilities. It is called *homogeneous* by Lippolt [10], since every storage location in the AS/RS has the same probability of being occupied or free. The pure moving times of S/R-machines to the storage location or during relocations are already known [11], therefore this paper focuses on relocation probabilities, number of expected relocations and movement times within the storage channels.

This paper is structured as follows. In chapter 2, a short literature review points out the necessity of further academic research in the topic of travel time models for multi-deep AS/RS and presents related work. Chapter 3 shows the basics of travel time calculations and explains the underlying assumptions and strategies on which this paper is based. Chapter 4 covers the work done by Lippolt [10] for double-deep AS/RS, revisiting and extending it to all necessary components for the travel time model presented in this paper. The travel time model for multi-deep AS/RS is introduced in chapter 5. This is followed by a discussion in chapter 6 about the developed model and strategies compared with state of the art models and strategies for double- and multi-deep AS/RS. Finally, the proposed travel time model is compared with a simulation for verification in chapter 7.

2. LITERATURE REVIEW

This paper focuses on the exact determination of expected travel times and therefore relocation probabilities and expected number of relocations during storage or retrieval operations. This is done via mathematical and analytical models. An alternative approach is to estimate travel times using simulations, which has been evaluated in the past for double-deep and multi-deep AS/RS [12] [13]. Fan et al. state that “multi-deep AS/RS has numerous variables as well as complex causations” and therefore the, “simulation method should be the preferred method” [13]. However, simulation models have to be performed for all possible AS/RS layout options. In practice, a simulation has to be implemented for every new AS/RS; therefore, analytical models are more universally applicable. This literature review focuses on different AS/RS variations such as vehicle AS/RS, flow-rack AS/RS, push-back AS/RS or conveyor based AS/RS [14]. All of these consider mutli-deep storage and, depending on the AS/RS design, the necessity of relocations. Hence, the methods to calculate relocation probabilities for one type of AS/RS may then be transferred to other AS/RS types.

The basis of analytical travel time models was laid by Gudehus [1] and later Bozer and White [11][15], who outlined the travel times for single-deep AS/RS serving one rack and a random storage and retrieval strategy. They use a continuous pick face to determine the expected average travel time for a storage or retrieval operation in closed-form expressions (see Chapter 3). The next step was the extension of this basic model to double-deep AS/RS by Lippolt [10], who provided an exact relocation probability depending on the stock filling level z, assuming the same strategies. This means, that the relocation probability is only depending on the number of stored goods compared to the total capacity of the AS/RS, which is possible since the random storage and retrieval operations lead to a dynamically stable storage channel allocation with empty, half full and full channels. Lerher et al. [16] deviate from the random storage strategy and introduce a travel time model for double-deep AS/RS assuming, that the first storage lane is filled completely before the second storage lane gets used at all. The same principle is applied to the travel time calculation of double-deep shuttle-based AS/RS [17]. This assumption leads to mild deviations between simulations and analytical models, because there can be situations in which the second storage lane is occupied and other storage channels are empty.

For multi-deep AS/RS, there are two categories of related literature available. The first category deals with relatively simple assumptions, which eliminate the relocation necessity or ensure a constant number of necessary relocations independently from the stock filling level. The most basic works available do not regard relocations [6][18], or assume that these are not necessary at all due to batching of storage goods [5]. Furthermore, several authors assume an AS/RS, which is completely filled and therefore, whenever a storage good is blocked the maximal number of relocations is required. This way, the calculation of the number of relocations only depends on the location of the affected storage good in the storage channel[8][9]. Starting with de Koster et al. [4], a new type of AS/RS was introduced which completes the necessary relocations within the rack itself. Therefore, this approach needs different methods to calculate the relocation probabilities, which are not useful for this paper. However, this assumption is widely used as in e.g. [7][19][20].

All papers mentioned in the first category calculate the travel times independently from the stock filling level z, in contrast to the papers of the second category which include the dependency from z. Sari et al. [3] introduce an AS/RS where one S/R machine fills the racks and another S/R machine retrieves storage goods from the other side of the rack. Relocations are realized via a separate conveyor belt. Based on the stock filling level, Sari et al. determine the relocation probability, however Sari et al. [3] also assume a minimal variance of storage channel fillings, which leads to a different relocation probability for double-deep AS/RS compared to the analytically derived results of Lippolt et al. [10]. Comparing simulations with analytical models, the same deviations appear as described for [16] and [17]. However, this approach has been used in the last decade by several authors, for example Ghomri et al. [21]. Eder [22] determines the relocation probability depending on the stock filling level and tries to determine the relocation probability based on an equally distributed storage channel allocation, as used in this paper. This means, that every storage location has the same probability to be occupied. However, this assumption is contrary to the random storage and retrieval strategy used by Eder [22][23], which is shown in [10] for a double-deep AS/RS. The different assumptions and consistencies in application of assumptions lead to different models for the calculation of relocations as shown in chapter 6. This paper presents a storage and retrieval strategy, with which the equally distributed storage channel allocation is achievable.

The academic research in the second category has in common, that the relocation probabilities are determined relative to the stock filling level. So far, the assumptions made in these papers are not consequently implemented, which leaves logical inconsistencies, thus differences between simulation and analytical model. This paper resolves the problem by deriving a travel time model relative to the stock filling level, using the equally distributed storage channel allocation consequently, also used by Eder [22][23]. Additionally, a random strategy is applied for storage and retrieval operations, but relocation operations follow a predefined algorithm.

To sum up, the available academic research for travel times models of multi-deep AS/RS with relocations, dependent on the stock filling level, is based on Bozer and White [15] and tries to transfer the basics for single-deep AS/RS to multi-deep AS/RS. It is ignored, that the random assignment of storage locations to new storage goods leads to very heterogeneous storage channel allocations. Even with a high stock filling level, there are empty storage channels, which might not be desirable in practice, however a random storage assignment strategy should regard this, which is the purpose of this work. Furthermore, it is shown how a equally distributed storage channel allocation can be realized with the help of a special relocation algorithm.

Further information on existing travel time models for AS/RS can be found in a review paper from Azadeh et al. [14].

3. TRAVEL TIME BASICS FOR AS/RS

The travel time model presented in this paper is based on the most basic version of an AS/RS: one S/R machine serves each aisle, which cannot leave this aisle and which can transport only one storage unit at a time [2]. To simplify the travel time model, only one aisle and one rack, parallel to the rails of the S/R machine is covered. This rack has an extension of X storage channels horizontally, Y storage channels vertically and a channel depth of n. Every rack has $X \cdot Y$ storage channels and $X \cdot Y \cdot n$ storage locations. Furthermore, the following assumptions apply in this paper and for the basics of travel time calculation:

- Storage racks are 'considered to be a continuous rectangular pick face' where the input and output point (I/O point) is located in the 'lower left-hand corner' of the rack [16]
- The S/R machine is equipped with a telescopic arm, which allows to reach not only the first storage position in a storage channel but also deeper storage positions. This channel drive has the same velocity in both directions
- The S/R machine can perform the vertical and horizontal move simultaneously and with such speed, that the full length and height l and h gets reached at the same time. Bozer and White call this characteristic 'square in time' [11]
- The acceleration of the S/R machine is not regarded in this paper and therefore, the speed of the S/R machine is constant
- The dwell point for all new command cycles is the I/O point

- There is no batching regarded in this work, all storage goods are assumed to be unique
- Random storage assignment and retrieval strategies are used
- The probability for a storage operation and for a retrieval operation are the same

In general storage and retrieval operations can be integrated in single command cycles and dual command cycles. During a single command cycle, only one storage or one retrieval operations is carried out and a dual command cycle combines both one storage and one retrieval operation. Hence, the S/R machine does not have to move to the I/O-point between the operations, which saves time. For all operations and calculations a unified notation is needed, which is presented in Table 1.

C_i	Storage channel state i
C_{ij}	Storage location j in a storage channel C_i
k	number of storage goods in a storage channel
n	Depth of channels
$P(Relo_n)$	Probability of a relocation during a storage or retrieval operation in a n-deep AS/RS
S; R	Storage or retrieval operation
t_A	Average movement time of the S/R machine from the I/O-point to the storage/retrieval channel and vice versa
$t_{C,n}$	Channel time, the telescopic arm of the S/R machine needs to perform during a storage or retrieval operation in a n-deep AS/RS
$t_{C\ Relo,n}$	Movement time, the telescopic arm of the S/R machine needs to move into a channel during a relocation operation in a n-deep AS/RS
t_d	Dead time – e.g. reaction time or sensor response time – which is constant for all command cycles [24]
$t_{DC,n}$	Average travel time of a dual command cycle in a n-deep AS/RS
t_E	Average movement time of the S/R machine from the storage/retrieval channel to the relocation channel
t_F	Average movement time of the S/R machine from one storage/retrieval channel to another storage/retrieval channel
t_h	Handling time of the S/R machine for the picking process of a storage good
$t_{ReloS,n}$; $t_{ReloR,n}$	Average relocation time during a storage or retrieval operation in a n-deep AS/RS.
$t_{S,n}$; $t_{R,n}$	Average travel time of a single command storage/retrieval cycle in a n-deep AS/RS
t_{step}	Movement time the telescopic arm of the S/R machine needs to move one depth step into a channel
t_{XY}	Movement time of the S/R machine to move the entire horizontal and vertical distance of the AS/RS
v_x; v_y	Velocity of the S/R machine horizontally/ vertically
X; Y	Number of storage channels horizontally/ vertically
z	Stock filling level
α_n	Number of steps the S/R machine has to move into a channel during a storage or retrieval operation in a n-deep AS/RS
$\alpha_{Relo,n}$	Number of steps the S/R machine has to move into a channel during a relocation operation in a n-deep AS/RS
β_n	Expected number of relocations during a storage or retrieval operation in a n-deep AS/RS.

Table 1: Notation for the travel time calculation.

A single command storage cycle for a single-deep AS/RS consists of the pick up time for the storage good at the I/O-point, the movement to the storage point, the channel time the telescopic arm of the S/R machine needs to reach the final storage location, putting down the storage good, the channel time retracting the telescopic arm and the movement time back to the I/O-point. The dead time is added on top of the described operation. The travel time for a single command retrieval cycle can be calculated similarly.

$$t_{S,1} = t_{R,1} = 2 \cdot t_h + 2 \cdot t_A + 2 \cdot t_{C,1} + t_d \tag{1}$$

t_h and t_d are constant and solely depending on the mechanics of the S/R machine. For a single-deep AS/RS, $t_{C,1} = \alpha_1 \cdot t_{step}$ is constant and only depending on the mechanics of the telescopic arm with $\alpha_1 = 1$ due to the single depth of the AS/RS. The movement times t_A and t_B do also depend on the location of the storage channel, which is randomly distributed over all free storage channels. Bozer and White developed the baseline travel time model for the movement of the S/R machine, which is able to determine the average expected movement time to a randomly chosen storage channel in the rack [11]. This methods enables the travel time calculation based on one single representative storage channel. The movement times for this representative storage channel are:

$$t_A = \frac{2}{3} \cdot t_{XY} \tag{2}$$

The trivial travel time presented in (1) is only valid for single-deep AS/RS. When AS/RS with greater depths are introduced, relocations can be necessary due to blocking storage goods. These relocation times $t_{ReloS,n}$ and $t_{ReloR,n}$ have to be added to (1). $t_{ReloR,n}$ is composed of the pick up time for the blocking storage good, the channel times for the telescopic arm at the retrieval channel, the movement time to the relocation channel, the channel times at the relocation channel, the put down time at the relocation location and the movement time back to the retrieval channel. This procedure has to be done for every relocation. Chapter 5.5 shows that the channel times during a relocation are the same for storage and retrieval and also the same in the storage or retrieval and relocation channel.

$t_{ReloS,n}$ has basically the same structure as $t_{ReloR,n}$, but an additional term has to be considered. During a relocation, the S/R machine performs an empty run from the I/O point to the storage channel t_A. This is due to the fact that a single command cycle starts at the I/O point and the S/R machine can only do a relocation if it is empty. The relocation itself occurs in the same manner as during a retrieval operation. Afterwards, the S/R machine returns to the I/O point t_B to pick up the storage good, which has to be stored. Due to the fact, that the S/R machine moves to the I/O point after the last relocation, the last movement between relocation channel and storage channel does not take place. This correction term is only necessary if a relocation occurs, with the relocation probability $P(Relo_n)$.

$$t_{ReloS,n} = (2t_h + 4t_{CRelo,n} + 2t_E) \cdot \beta_n + (t_B + t_A - t_E) \cdot P(Relo_n) \quad (3)$$

$$t_{ReloR,n} = (2t_h + 4t_{CRelo,n} + 2t_E) \cdot \beta_n \quad (4)$$

$t_{CRelo,n}$, t_E, $P(Relo_n)$ and β_n are still undetermined. While t_E has already been determined by Bozer and White [11], finding $t_{CRelo,n}$, $P(Relo_n)$ and β_n is part of this paper. Again we assume, that the relocation location is randomly chosen from all storage channels of a predetermined state as described in chapter 5.2. Therefore, the movement time between two random storage channels is needed:

$$t_E = \frac{7}{15} \cdot t_{XY}. \quad (5)$$

Combining (1) to (5), the total travel times for single command storage and retrieval cycles are:

$$t_{R,n} = 2t_h + \frac{4}{3}t_{XY} + (2t_h + 4t_{CRelo,n} + \frac{14}{15}t_{XY}) \cdot \beta_n + 2t_{C,n} + t_d \quad (6)$$

$$t_{S,n} = t_{R,n} + P(Relo_n) \cdot \frac{13}{15} \cdot t_{XY} \quad (7)$$

A dual command cycle combines the two different single command cycles and reduces the movements from the I/O point to the storage channel and inversely. Additionally the S/R machine has to perform one movement from the storage to the retrieval channel, which is a movement between two random channels with $t_F = t_E = \frac{7}{15}t_{XY}$. The travel time of a dual command cycle is:

$$t_{DC,n} = 4t_h + \frac{9}{5}t_{XY} + (4t_h + 8t_{CRelo,n} + \frac{28}{15}t_{XY})\beta_n + 4t_{C,n} + t_d + P(Relo_n)\frac{13}{15}t_{XY} \quad (8)$$

4. TRAVEL TIME MODEL FOR DOUBLE-DEEP AS/RS

The parameters $t_{C,n}$, $t_{CRelo,n}$ and β_n in (8) are not known and are derived in this chapter for a double-deep AS/RS. The determination of these parameters is influenced by several further strategies:

- Storage strategy – At which empty storage locations can a new storage good be stored?
- Retrieval strategy – Which storage good is chosen to be retrieved?
- Storage channel states – Where can storage goods be stored inside a storage channel depending on other storage goods inside this storage channel?
- Relocation strategy – At which empty storage location can a blocking storage good be relocated?

These four characteristics are explained based on a double-deep AS/RS and can be applied to any depth afterwards. The storage strategy is random in this paper, as mentioned in chapter 3. This means, that the storage location is chosen randomly under all free storage locations. The retrieval strategy is random as well, which means that all stored goods have the same probability to be retrieved. All valid storage channel states in a double-deep AS/RS are presented in Figure 1.

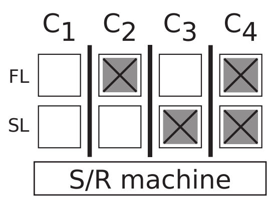

Figure 1: All possible storage channel states in a double deep AS/RS with a homogeneous allocation structure. Occupied storage locations shaded in grey.

Based on Lippolt [10], two different storage allocation strategies are possible:

- *homogeneous allocation structure* – All channels shown in Figure 1 are valid channel options
- *stack allocation structure* – Channel C_3 is not allowed, because this storage channel causes relocations during the storage operation, when the empty storage location at the wall is chosen for storage operation

In this paper, the *homogeneous allocation structure* is chosen, since in combination with special relocation strategies travel times can be modelled conveniently in closed form expressions. For low stock filling levels, the *homogeneous allocation structure* has the advantage over the *stack allocation structure*, that the S/R machine does not have to execute the maximal number of channel drives, but can put the storage good at the storage location closest to the aisle. On the opposite, for high stock filling levels, the *stack allocation structure* has the advantage, that during storage operations no relocations are necessary.

Before the relocation strategy can be explained, all storage and retrieval operations must be described. This will be based on Figure 2.

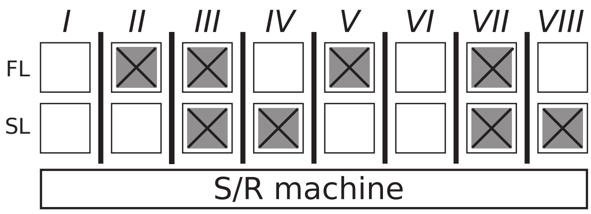

Figure 2: Example of a double-deep AS/RS. FL = First Lane; SL = Second Lane.

Figure 2 shows a double-deep AS/RS with eight storage channels I to $VIII$, which are from storage channel type C_1, C_2, C_3 and C_4. In total, eight different storage (S) and retrieval (R) operations are possible in a double-deep AS/RS:

- S_1: The storage good is stored into channel I at the first lane → from C_1 to C_2
- S_2: The storage good is stored into channel I at the second lane → from C_1 to C_3
- S_3: The storage good is stored into channel II at the second lane → from C_2 to C_4
- S_4: The storage good is stored into channel IV at the first lane. Therefore, the storage good in the second lane has to be relocated into channel II or V → from C_3 to C_4
- R_1: The storage good is retrieved from channel II from the first lane → from C_2 to C_1
- R_2: The storage good is retrieved from channel IV from the second lane → from C_3 to C_1
- R_3: The storage good is retrieved from channel III from the second lane → from C_4 to C_2
- R_4: The storage good is retrieved from channel III from the first lane. Therefore, the storage good in the second lane has to be relocated into channel I or VI → from C_4 to C_3

With the predetermined relocation channels, only one channel changes its state during a storage or retrieval operation. Lippolt concludes, that the *homogeneous allocation structure* leads to a stable system where every storage location has the probability z that it is occupied and $1 - z$ that the storage location is free [10]. z is the stock filling level and can be calculated by:

$$z = \frac{\text{Number of storage goods in the AS/RS}}{X \cdot Y \cdot n}.$$

This results in the following probabilities, that a random storage channel is of type C_1, C_2, C_3 or C_4 [10].

$$P(C_a) = (1-z)^2, \quad P(C_b) = (1-z) \cdot z,$$
$$P(C_c) = (1-z) \cdot z, \quad P(C_d) = z^2$$

The probabilities for the eight storage and retrieval operations are:

$$P(S_1) = P(R_1) = \tfrac{1-z}{2}, \quad P(S_2) = P(R_2) = \tfrac{1-z}{2},$$
$$P(S_3) = P(R_3) = \tfrac{z}{2}, \quad P(S_4) = P(R_4) = \tfrac{z}{2}.$$

With this the number of relocations and the relocation probability can be determined to

$$\beta_2 = P(Relo_2) = \frac{z}{2}. \tag{9}$$

4.1. Total Travel Times of Single and Dual Command Cycles

Following (6) to (8) and (9), only the determination of $t_{C,n}$ and $t_{CRelo,n}$ remains undefined to determine the single and dual command cycle times.

Figure 2 shows that during S_1 the S/R machine has to move to the second lane and afterwards to the first lane. This means it performs two steps inside the storage channel towards the wall and backwards. For S_2 and S_3 the S/R machine only has to move to the second lane, which counts as one step. Excluding the relocation operation during S_4 – which is discussed in the following – the S/R machine executes two steps during S_4. The channel travel time during a storage operation t_{C_S} is the sum of the probabilities of all steps of all storage operations divided by the sum all storage operations and $t_{C_S} = t_{C_R}$:

$$t_{C,2} = \frac{2P(S_1) + P(S_2) + P(S_3) + 2P(S_4)}{\sum_{i=1}^{4} P(S_i)} t_{step} = \frac{3}{2} t_{step}.$$

For a double-deep AS/RS, $t_{CRelo,2}$ is the same for S_4 and R_4. As shown in Figure 2, during a relocation the S/R machine has to execute only one step in a storage channel and therefore $t_{CRelo,2} = t_{step}$.

Even tough relocation operations presuppose that the relocation channel is not randomly chosen from all channels, the average travel time to a random channel of the predetermined type of channels is equal to (5). This is valid, because all channels of the same type are equally distributed over the AS/RS. In total $t_{S,2}$, $t_{R,2}$ and $t_{DC,2}$ in a double-deep AS/RS are:

$$t_{S,2} = 2t_h + \frac{4}{3}L + (2t_h + 4t_{step} + \frac{27}{15}L)\frac{1}{2}z + 3t_{step} + t_d$$
$$t_{R,2} = 2t_h + \frac{4}{3}L + (2t_h + 4t_{step} + \frac{14}{15}L)\frac{1}{2}z + 3t_{step} + t_d$$
$$t_{DC,2} = 4t_h + \frac{9}{5}L + (4t_h + 8_{step} + \frac{41}{15}L)\frac{1}{2}z + 6t_{step} + t_d$$

5. TRAVEL TIME MODEL FOR MULTI-DEEP AS/RS

As presented for double-deep AS/RS in chapter 4, the storage and retrieval strategy is random. For the calculation of the single and dual command cycle travel times presented in 6, 7 and 8, the relocation probability, number of relocations and channel times have to be defined for a multi-deep AS/RS. Therefore, chapter 5.1 determines the different channel states depending of the depth of the AS/RS and their probabilities. Based on these probabilities, storage and retrieval operations are determined in chapter 5.2. Chapter 5.3 proves, that the derived probabilities are correct and lead to a stable system state with a *homogeneous allocation structure*. Afterwards, the relocation probabilities, number of relocations, channel times and total travel times are derived.

5.1. Storage channel state probabilities for multi-deep AS/RS

The set of all channel states is given by the n-ary Cartesian product of $\{0, 1\}$, where 1 indicates that the storage location is occupied and 0 indicates that the storage location is free. The number of different channel configurations is 2^n.

The $2^n \times n$ matrix C is used to describe the possible channel states, where each row C_i corresponds to a unique channel state. Every C_{ij} can be interpreted as a storage location, which is either occupied ($C_{ij} = 1$) or empty ($C_{ij} = 0$). This means, that $C_{ij} = 1$ can also be interpreted as the storage good at this location. For example for $n = 3$, the corresponding matrix reads as the following and can also be found in Figure 3, where every C_i is represented by its roman number:

$$C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Following the *homogeneous allocation structure*, the probability for each channel state C_i and the aggregate probability of obtaining a channel with k storage goods can be described with the following expressions.

$$P(C_i) = \prod_{j=1}^{n} (1-z)^{1-C_{ij}} \cdot z^{C_{ij}} \tag{10}$$

$$P(k) = \binom{n}{k} (1-z)^{n-k} \cdot z^k \tag{11}$$

Summing up all probabilities $P(C_i)$ trivially results in 1.

5.2. Storage and Retrieval Operations in a multi-deep AS/RS

Both storage and retrieval operations can be understood as a transition between two channel states C_i and $C_{i'}$. A storage operation S and a retrieval operation R can be defined as:

$$S : C_i \rightarrow C_{i'}, \quad R : C_{i'} \rightarrow C_i$$

Retrieval operations can be understood as an inversion of a storage operation. For each storage operation, there exists exactly one retrieval operation that does the opposite, by just reverting the channel state transition. In order to achieve the homogeneous allocation structure, a storage or retrieval operation needs to satisfy two principles:

1. Only one storage location C_{ij} in the channel C_i is allowed to change
2. This change must be either from $C_{ij} = 0$ to $C_{i'j} = 1$ at a storage operation or from $C_{ij} = 1$ to $C_{i'j} = 0$ at a retrieval operation

In total, there are $\sum_{k=0}^{n} \binom{n}{k} \cdot (n-k) = 2^{n-1} \cdot n$ different storage operations and the same number of retrieval operations. For further considerations, it is important to know the probability, that a storage or retrieval operation affects a certain channel state. The conditional probability of a storage or retrieval operation taking place in a channel state C_i is

$$P(C_i|S) = \frac{P(C_i)}{\sum_{k=0}^{n} (n-k) \cdot P(k)},$$

$$P(C_i|R) = \frac{P(C_i)}{\sum_{k=0}^{n} k \cdot P(k)}.$$

Not all operations S and R for any C_{ij} can be executed directly, due to blocking storage goods. Relocation operations are necessary if $\sum_{d=1}^{n} C_{ij+d} \geq 1$. The necessary number of relocations is then the numbers of goods stored in front of C_{ij}.

The basic idea of relocations is, that the blocking storage goods are not reallocated randomly, but to a storage channel in a predetermined channel state. The relocation operation is exemplary shown by the Figures 3, 4 and 5. When the storage good i has to be retrieved, the *ii* and *iii* storage goods must be relocated.

A storage channel C_i for the relocation of a blocking storage good has to be found which fulfills the following conditions:

1. During a retrieval operation, the lane of the storage good to be retrieved must be empty. In a storage operation it must be occupied
2. All other storage locations C_{ij} in channel C_i with a lower lane than the relevant storage or retrieval location (storage good *i* in Figure 3) must be the same as in the original channel
3. All storage locations C_{ij} in channel C_i with the same or a higher lane than the relevant storage or retrieval location must be empty

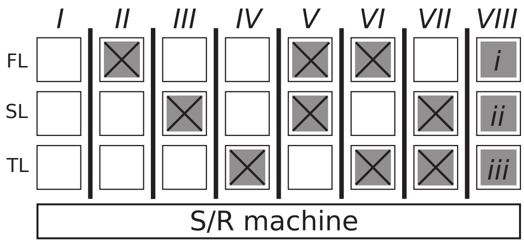

Figure 3: Triple-deep AS/RS. Storage good i shall be retrieved out of channel III.

Only one storage channel type fulfills these conditions, which is channel *III* for the first relocation. After the first relocation the AS/RS looks as shown in Figure 4. The second relocation follows the same three rules and the result is shown in Figure 5. After this relocation, the storage good *i* can be retrieved. It is also noticeable, that the S/R machine has to perform the same number of steps inside the original and the relocation channel for every relocation.

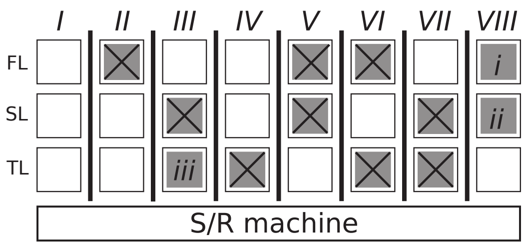

Figure 4: Triple-deep AS/RS after the first relocation.

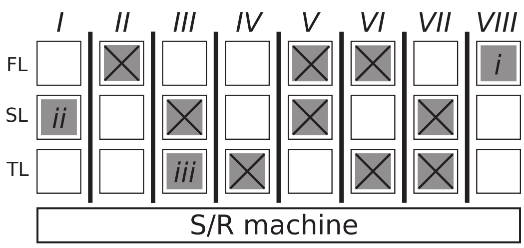

Figure 5: Triple-deep AS/RS after the second relocation.

It should be emphasized that the described procedure for a relocation aims at keeping the number of channels from every type stable, except from the original channel. In total only one channel changed its state and this state transition is illustrated 6.

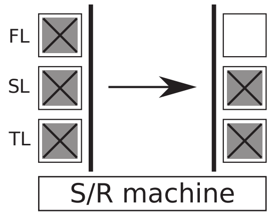

Figure 6: Channel state transition Triple-deep AS/RS after the second relocation.

5.3. Stable System State Proof

The hypothesis is, that the storage channel probabilities presented in (10) and (11) and the storage, retrieval and relocation operations explained in chapter 5.2 build a stable system of a multi-deep AS/RS.

This stable system requires, that every C_i can be transformed into any other $C_{i'}$ with one more or one less storage good by only one storage or retrieval operation. Furthermore it is necessary that every storage or retrieval operation is directly reversible. This means, that for every storage operation $C_i \rightarrow C_{i'}$, there is exactly one retrieval operation which reverses the storage operation $C_{i'} \rightarrow C_i$. Such a stable system can be described with a Markov chain.

Therefore, a finite state space as well as transition probabilities between these states must be defined. These states can be interpreted as the 2^n possible channel states. The transition probabilities are the conditional probabilities $P(S|C_i)$ and $P(R|C_i)$, that a storage or retrieval operation is performed regarding a channel C_i. This Markov chain can be described with a $2^n \times 2^n$ transition matrix P. The entries in this matrix are the following:

- With every row of P and therefore every channel state, $(n-k)$ storage and k retrieval operations can be performed. These are expressed with the conditional probabilities $P(S|C_i)$ and $P(R|C_{i'})$. These are distributed to specfic C_{ij} at j, where storage or retrieval operations are possible as explained in chapter 5.2
- The entries of the main diagonal is then $1 - k \cdot P(R|C_i) - (n-k) \cdot P(S|C_i)$
- All remaining entries are $C_{ij} = 0$

This transition matrix can be resolved to:

$$\begin{pmatrix} P(C_1) \\ \dots \\ P(C_{2^n}) \end{pmatrix} = \begin{pmatrix} P(C_1) \\ \dots \\ P(C_{2^n}) \end{pmatrix}^{\top} \cdot P$$

It can now be shown that every equation of the equation system above is valid, by proving, that:

$$P(C_1) = P(C_1) \cdot (1 - n \cdot P(S|C_1)) + \sum_{i'=2}^{n+1} P(C_{i'}) \cdot P(R|C_{i'})$$

This can be resolved into n equations:

$$P(C_1) \cdot P(S|C_1) = P(C_2) \cdot P(R|C_2),$$
$$\ldots$$
$$P(C_1) \cdot P(S|C_1) = P(C_{n+1}) \cdot P(R|C_{n+1}).$$

Or more generally it has to be shown, that:

$$P(C_i) \cdot P(S|C_i) = P(C_{i'}) \cdot P(R|C_{i'}). \tag{12}$$

If this is achievable with the so far derived channel state probabilities and storage and retrieval probabilities, these probabilities make up a stable multi-deep AS/RS together. In the following the hypothesis will be proven for all storage operations. The prove for retrieval operations is then trivial.

$$P(C_i) = (1-z)^{n-k} \cdot z^k \tag{13}$$

$$P(C_{i'}) = (1-z)^{n-k-1} \cdot z^{k+1} \tag{14}$$

$$P(S|C_i) = \frac{P(C_i|S) \cdot P(S)}{P(C_i)} = \frac{P(S)}{(1-z) \cdot n} \tag{15}$$

$$P(R|C_{i'}) = \frac{P(C_{i'}|R) \cdot P(R)}{P(C_{i'})} = \frac{P(R)}{z \cdot n} \tag{16}$$

Filling (13) to (16) into (12) results in:

$$(1-z)^{n-k} \cdot z^k \cdot \frac{P(E)}{(1-z) \cdot n} = (1-z)^{n-k-1} \cdot z^{k+1} \cdot \frac{P(A)}{z \cdot n}$$

and

$$(1-z)^{n-k-1} \cdot z^k \cdot \frac{P(E)}{n} = (1-z)^{n-k-1} \cdot z^k \cdot \frac{P(A)}{n}$$

Following the assumption, that $P(E) = P(A)$, it is shown that (12) is true and the Hypothesis can be confirmed.

5.4. Relocation Probabilities and Number of Relocations

The relocation probability can be determined by calculating the average number of blocking storage goods for storage lane C_j. This is identical to the number of occupied storage locations k' in the $n - j$ storage locations in front of the storage lane C_j. Since the probability that a new storage good is stored in any of the j columns is the same, due to the *homogeneous allocation structure*, the outer sum is divided by n. This procedure is the same for storage and retrieval operations and therefore $P(Relo_n)$ is the same for storage and retrieval operations.

$$P(Relo_n) = \frac{1}{n} \sum_{j=1}^{n} \sum_{k'=1}^{n-j} \binom{n-j}{k'} \cdot z^{k'} \cdot (1-z)^{n-j-k'}$$

For an easier calculation, the counter probability – $1 - P$(no blocking good in front of the regarded C_j) – can be determined as well.

$$\begin{aligned} P(Relo_n) &= 1 - \frac{1}{n} \cdot \sum_{j=1}^{n} (1-z)^{n-j} \\ &= \frac{(1-z)^n + zn - 1}{zn} \end{aligned} \tag{17}$$

For all AS/RS with $n > 2$ it holds $P(Relo_n) \neq \beta_n$. β_n can be determined similar to $P(Relo_n)$ and the multiplication with k'. The *homogeneous allocation structure* enables that β_n is the same for storage and retrieval operations:

$$\begin{aligned} \beta_n &= \frac{1}{n} \sum_{j=1}^{n} \sum_{k'=1}^{n-j} k' \cdot \binom{n-j}{k'} \cdot z^{k'} \cdot (1-z)^{n-j-k'} \\ &= \frac{(n-1) \cdot z}{2} \end{aligned} \tag{18}$$

5.5. Channel Travel Times

The deeper the racks of an AS/RS are designed, the longer the telescopic arm of a S/R machine needs to reach to the storage locations. This means, that the channel times rise relative to the total travel time with the depth of the AS/RS. t_C can be determined relatively easy:

$$t_{C,n} = \alpha_n \cdot t_{step} = \frac{1+n}{2} \cdot t_{step} \tag{19}$$

This means, that the storage and retrieval machine has to move halfway into the storage channel during a storage or retrieval operation. Once again, the *homogeneous allocation structure* allows this simple definition. It also enables that $t_{C\,Relo,n}$ is the same for storage and retrieval operations. It applies: $t_{C\,Relo,n} = \alpha_{Relo,n} \cdot t_{step}$. However, $\alpha_{Relo,n}$ can be determined by analysing every C_j all storage locations in front of C_j and calculating the number of steps to be taken for every of these $C_{j'}$. This sum divided by β_n is the amount of average steps during a relocation:

$$\begin{aligned} \alpha_{Relo,n} &= \frac{\frac{1}{n} \sum_{j=1}^{n} \sum_{a=1}^{n-j} a \cdot t_C^{n-j} \cdot \binom{n-j}{a} \cdot z^a \cdot (1-z)^{n-j-a}}{\beta_n} \\ &= \frac{n+1}{3} t_{step} \end{aligned} \tag{20}$$

5.6. Total Travel Times of Single and Dual Command Cycles

With (6) to (8) and (17) to (20) it is possible to determine the single and dual command cycles solely depending on the S/R machine and AS/RS characteristics (t_h, t_d, t_{XY}, t_{step}, n), and the stock filling level z.

$$\begin{aligned} t_{S,n} = \; & 2t_h + \tfrac{4}{3} t_{XY} + (2t_h + \tfrac{4n+4}{3} t_{step} + \tfrac{14}{15} t_{XY}) \tfrac{n-1}{2} z \\ & + (1+n) t_{step} + t_d + \tfrac{13}{15} t_{XY} \cdot \tfrac{(1-z)^n + zn - 1}{zn} \end{aligned}$$

$$\begin{aligned} t_{R,n} = \; & 2t_h + \tfrac{4}{3} t_{XY} + (2t_h + \tfrac{4n+4}{3} t_{step} + \tfrac{14}{15} t_{XY}) \tfrac{n-1}{2} z \\ & + (1+n) t_{step} + t_d \end{aligned}$$

$$t_{DC,n} = 4t_h + \frac{9}{5}t_{XY} + 2(n+1)t_{step} + t_d + (4t_h + \frac{8n+8}{3}t_{step} + \frac{28}{15}t_{XY})\frac{n-1}{2}z + \frac{13}{15}t_{XY} \cdot \frac{(1-z)^n + zn - 1}{zn} \qquad (21)$$

6. COMPARISON WITH STATE OF THE ART STRATEGIES

The *homogeneous allocation structure* leads to relocations during storage operations. This is different from state of the art strategies as presented by Eder [23] and Lippolt [10] and makes a comparison of the key aspects necessary. Eder presents a strategy where relocations only occur during retrieval operations, with the assumptions, that every storage location is filled with probability *z* and the *stack allocation structure* is applied. Lippolt on the other hand deduces the channel probabilities for double-deep AS/RS with a Markov chain, which results in different storage channel probabilities compared to Eder's assumption. We do assume the same channel probabilities as Eder, but show that random storage and retrieval strategies are not purposeful to achieve a consistent strategy.

To sum up, the model developed by Lippolt applies the most realistic strategy and adheres to the assumptions.

So far, it is not possible to transfer this approach to multi-deep AS/RS. Eder and this paper make different assumptions regarding strategy and modelling to achieve a model for multi-deep AS/RS. In a first step, these two models are compared with Lippolt for double-deep AS/RS and in a second step, the comparison between Eder and this paper for multi-deep AS/RS follows. The number of relocations – β – can be specified for a storage (S) and relocation operation (R):

$$\beta_{\text{Eder}} = \begin{cases} 0 & S \\ \sum_{i=0}^{n-2}\sum_{j=1}^{n-1-i} \frac{j}{n-i}\binom{n}{i} \cdot z^{n-i} \cdot (1-z)^i & R \end{cases}$$

$$\beta_{\text{Lippolt}} = \begin{cases} 0 & S \\ \frac{z}{1+z} & R \end{cases}$$

$$\beta_{\text{this paper}} = \begin{cases} \frac{(n-1)\cdot z}{2} & S \\ \frac{(n-1)\cdot z}{2} & R \end{cases}$$

Figure 7 shows, that Eder's model underestimates the relocation effort during a retrieval operation in a double-deep AS/RS by $\beta_{\text{Lippolt}} - \beta_{\text{Eder}} = \frac{2z-z^2-z^3}{2z+2}$ compared to the model of Lippolt. The model developed in this paper underestimates the relocation effort during a retrieval operation as well but only by $\frac{z-z^2}{2z+2}$. This means, compared to Eder, the model presented in this paper underestimates the relocation effort less by $\frac{z-z^2}{2} \geq 0$ for $0 \leq z \leq 1$.

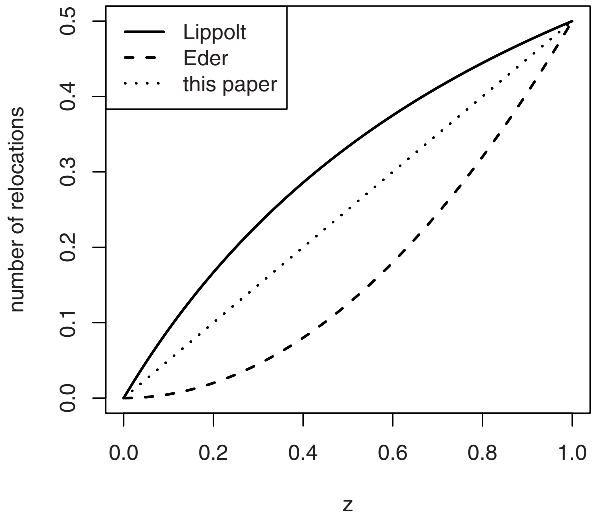

Figure 7: Comparison of three different models for the number of relocations β during a retrieval operation for double-deep AS/RS; Lippolt [10], Eder [23] and this paper.

For triple- and five-deep AS/RS, Figure 8 shows that Eder's model results in lower relocation efforts than our model during retrieval operations. Combining Figures 7 and 8, it is likely, that both models underestimate the relocation effort.

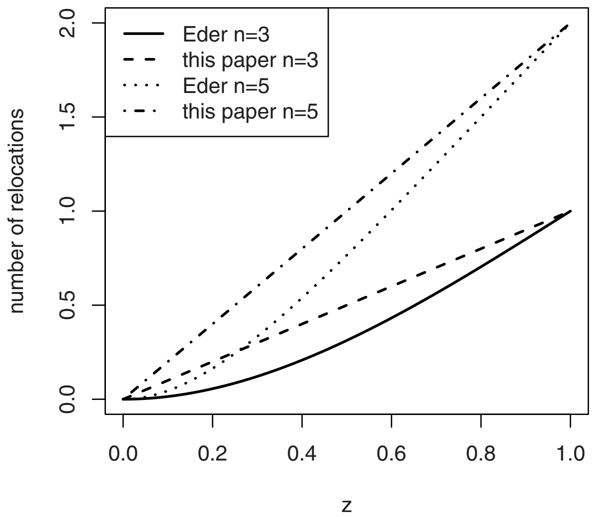

Figure 8: Comparison of two different models for the number of relocations β during a retrieval operation for multi-deep AS/RS (n = 3 and n = 5); Eder [23] and this paper.

Note that the relocation effort during a dual command cycle is double the effort presented in Figures 7 and 8 for the model presented in this paper, which overestimates the relocation effort compared to Lippolt. To sum up, common strategies for multi-deep AS/RS – such as Eder [23] – underestimate the relocation effort during retrieval operations and could be improved. A combination between the common models without relocations during storage operations and our model with higher relocations during retrieval operations could lead to a more accurate model.

7. SIMULATION

Via simulation it is shown, that the derived closed-form expression for dual command cycles is valid. The focus of the simulation is thereby on the verification of formula (21) and its components – especially the relocation probability and the number of relocations. It is not necessary to to show the homogeneous allocation, since this has been proven in chapter 5.3.

A Python based program simulates the procedure of a dual command cycle in an AS/RS, which follows the assumptions introduced in chapter 3 and the following:

1. The AS/RS follows the *homogeneous allocation structure*, which means that every storage location is occupied with the probability z or free with the probability $1 - z$
2. The position of storage channels of different channel states in the rack is uniformly distributed. This means, that every storage channel has the same probability to be in a certain channel state

A single run of the simulation reproduces one dual command cycle. The S/R machine selects a random storage location, executes relocations if necessary, picks the storage good at the I/O point and moves to the related storage channel and completes the storage operation. Afterwards, a storage good is chosen to be retrieved, the S/R machine moves to the retrieval channel, executes relocations if necessary, picks the storage good and moves to the I/O point. The number of relocations is determined by a sequence of Bernoulli experiments, this means a random experiment for each storage position in front of the determined storage or retrieval position is conducted, where either the storage position is occupied with a probability of z, or is not occupied with a probability of $(1 - z)$.

In total, the simulation analysis four different AS/RS, which are presented in Table 2. The first two AS/RS represent an practice-oriented AS/RS and the last two AS/RS especially show the behaviour in a large AS/RS. For all simulations, z was increased in 5% intervals from 5% to 95%. For each value of z for every AS/RS, one million dual command cycles were simulated, which results in 18 million total simulation runs.

For small rack sizes the movement time expressions (2) and (5) used so far, induce a significant error, since those assume a continuous rack, although it is discrete in reality. While the effect is negligible for large racks, as shown in the simulation, we need to adjust the movement times for the small rack case. The exact, non-continuous movement time expressions are:

$$t_A = \frac{1}{|X| \cdot |Y|} \sum_{x \in X} \sum_{y \in Y} \max\left\{\frac{x}{v_x}, \frac{y}{v_y}\right\} \tag{22}$$

$$t_E = \frac{1}{|X| \cdot |Y|} \sum_{x,x' \in X} \sum_{y,y' \in Y} \max\left\{\frac{|x - x'|}{v_x}, \frac{|y - y'|}{v_y}\right\} \tag{23}$$

In the figures 9 and 10, the simulation results and especially the relocation probabilities, number of relocations, travel time and the respective relative error are presented. The relative error is the difference between the simulation and the closed-form result divided by the closed-form result. The key findings of the simulations are:

- For low z, the relative errors for the relocation probability and the number of relocations is higher than for high z. This results out of the calculation method for relative errors, where small numbers are divided. In total the relative errors are < 1% and < 0.25% for practice-oriented $z > 0.5$.
- For a large rack the relative error of the travel time is less than 0.15%. This confirms the analytically derived travel time model of chapter 5.6.
- For a small rack the relative error of the travel time is less than 0.15%, when using the discrete movement time expression.

	1.AS/RS	2.AS/RS	3.AS/RS	4.AS/RS
X	100	100	10,000	10,000
Y	10	10	2,000	2,000
n	3	5	3	5
t_h	10	10	10	10
t_d	5	5	5	5
t_{XY}	10	10	100	100
t_{step}	1	1	1	1

Table 2: Characteristics of the four example AS/RS.

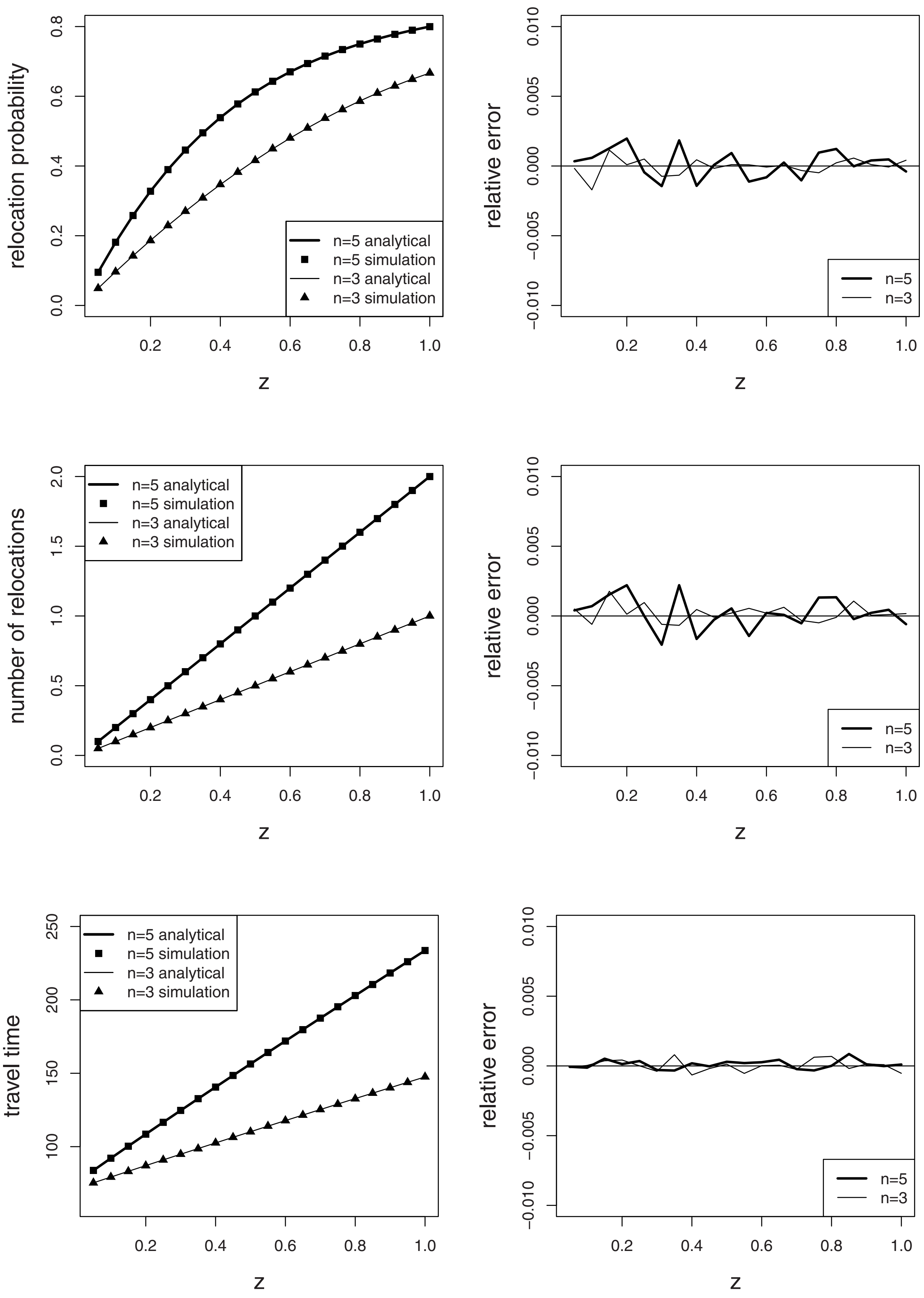

Figure 9: Simulation results for the 1. and 2. AS/RS of table 2.

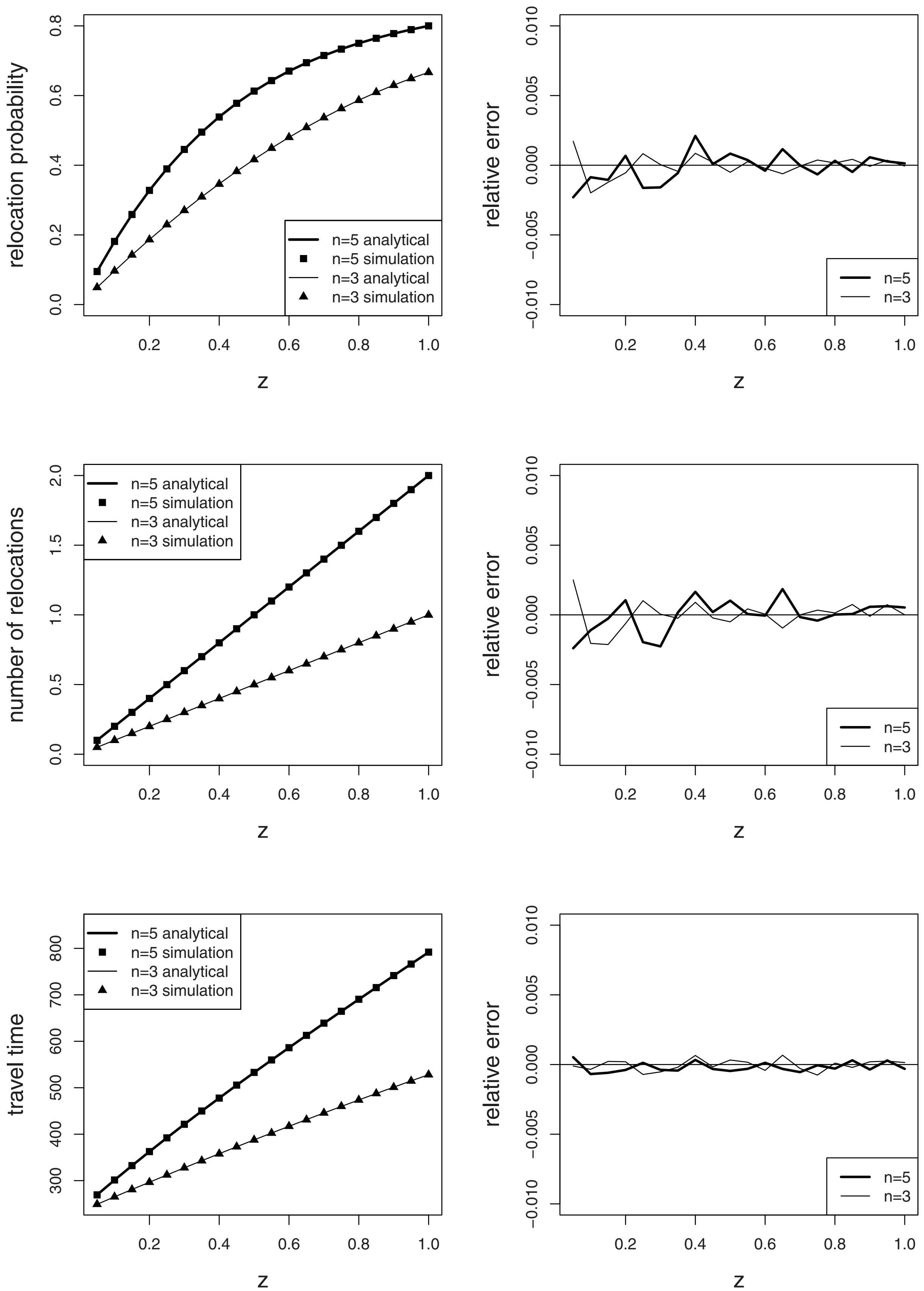

Figure 10: Simulation results for the 3. and 4. AS/RS of table 2.

8. CONCLUSION

The *homogeneous storage allocation* offers an approach to determine the relocation probability, number of relocations and single and dual command cycle times for multi-deep AS/RS. In this paper, it is accomplished to derive the mentioned characteristics analytically for the first time while strictly following the random storage assignment and retrieval strategy. The *homogeneous storage allocation* is proved in this paper and the other components of the travel time model are verified via simulation. In this simulation, the relative error of the dual command cycle travel time for large AS/RS is below 0.1% and for practice-oriented AS/RS below 0.1%.

The focus of this paper is the determination of relocation probabilities and number of relocations as well as the travel times in the storage channels itself during storage, retrieval and relocation operations. The already existing knowledge about acceleration of the S/R machine or different storage and retrieval strategies, such as shortest paths, are not part of this work to not unnecessarily over complicate the travel time models and focus on the key aspects; moreover, the mentioned aspects do not have any impact on the relocation probability and number of relocations. The key findings can be adapted to other existing travel time models such as flow-rack AS/RS [3] or shuttle-based AS/RS [5]. Especially the relocation probability, number of relocations and thus the relocation time can be updated in the existing models, which could result in lower error rates comparing the analytical models with simulations in the existing related academic research.

Since this work is based on the assumption of a *homogeneous allocation structure*, relocations during a storage operation may be necessary. This is not desirable, but shows how an equally distributed storage channel allocation can be achieved. Neither this paper nor the state of the art literature give a satisfying model with a combination of true random storage and retrieval strategies and a real storage channel probability distribution. However, the model presented in this work underestimates the real relocation effort less than comparable state of the art models, which advances the travel time research. Furthermore, the findings in this work can be considered as a possible upper limit for the travel times for stock filling levels in a practice-oriented range in AS/RS with a *stack allocation structures*.

Future research on this topic should include an analytical model for *stack allocation structures* including the relocation probability, number of relocations and channel travel times. Furthermore it is worth investigating different storage assignment strategies for *stack allocation structures*. Also the travel times of the *homogeneous allocation structure* could be reduced by regarding topics like ABC-zoning or batching of storage goods.

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