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Supply Chain Network Design under Uncertainty with Evidence Theory

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ABSTRACT

In this paper, we present a new approach to design a multi-criteria supply chain network (SCN) under uncertainty. Demands, supplies, production costs, transportation costs, opening costs are all considered as uncertain parameters. We propose an approach based on evidence theory (ET), analytic hierarchy process (AHP) and two-stage stochastic programming (TSSP). First, we integrate ET and AHP in order to include several criteria (social, economical, and environmental) and the uncertain experts decisions for selecting the best set of facilities. Second, we combine evidential data mining and TSSP approach: (i) to design the SCN, (ii) to take into account the uncertainty of supply chain parameters, and (iii) to reduce scenarios number by retaining only the significant ones. Finally, we illustrate the model with computational study to highlight the practicality and the efficiency of the proposed method.

KEYWORDS Supply chain design · Two-stage Stochastic Programming · Evidence theory · Evidential data mining · BF-AHP

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1. INTRODUCTION

The most challenging strategic issues that face business organizations today are in the area of Supply Chain (SC). SC is a network of suppliers, factories, warehouses, and distribution centers through which raw materials are procured, transformed, and delivered to customers. The success of a SC depends on the most basic decision of SC management, which influences all other decision levels. The topic of designing a SC is called supply chain network design (SCND). According to Chopra and Meindl [10], SCND problem comprises the decisions regarding the number and location of production facilities, the capacity of each facility, the assignment of each market region to one or more locations, and supplier selection for sub-assemblies, components and materials. Extensive reviews surveying various issues in this area are available, for example, in [19]. Indeed, building a sustainable supply chain nowadays has become the ultimate objective of intelligent organizations, where the goal is not only to minimize common costs, but also to integrate multi-criteria in the SCND and to reduce vulnerability due to uncertainty, by reducing possible sources of losses [19, 30, 35].

In this paper, we deal with the design of a multi-criteria SCN under uncertain environment in order to satisfy customers and to respect environmental, social, and economical requirements. We consider a multi-criteria, multi-level, single product, uncertain SC parameters and single period SCND problem. The network has three levels: suppliers, production plants and customers. We propose a new approach based on evidence theory (ET), analytic hierarchy process (AHP) and two-stage stochastic programming (TSSP) to design a SCN under uncertainty. In this work, our most significant contributions are the following:(i) we integrate experts decisions uncertainty, environmental, social and economical aspects in the SCND problem using the BF-AHP method, (ii) we apply the ET, which is a strong formalism in modelling uncertainties, to define the most relevant scenarios that can be used in the TSSP model. The set of scenarios are mined from an expert opinion database with evidential data mining, (iii) we conduct a comprehensive set of numerical studies. Consequently, we reach some useful managerial insights.

The rest of the paper is organized as follows. In section 2, we detail the literature review. In section 3, the basics of ET are highlighted from the transferable belief model interpretation [41]. In section 4, the new

based belief SCND approach is detailed. Section 5 includes application of the proposed model and offers an analysis of the computational results. Finally, conclusions are drawn and perspectives are discussed in section 6.

2.   LITERATURE REVIEW

The business environment under which a SCN operates is unknown and critical parameters such as customer demands, costs, quantities, and capacities are uncertain. The importance of uncertainty in SCND has encouraged researchers to handle stochastic parameters in SC problems [9, 20, 49]. Various modelling approaches have been developed to deal with this complex problems, such as probability theory, fuzzy sets theory [51], possibility theory [7, 50], and the evidence theory [37].

In the area of stochastic SCND problems, scenario-based approaches such as TSSP result in more models [2, 4, 35, 42, 45, 46]. Generally, in these studies, the first stage decisions belong to the strategic decision level, and second stage decisions are tactical and operational decisions are made. For instance, Mir Hassani et al. [24] presented a TSSP model to design a SCN with uncertain demand scenarios. The first stage decisions concern the location of plants and distribution centers and the setting of their capacity levels. The second stage decisions concern the optimization of production and distribution quantities. Santoso et al. [35] proposed a TSSP for a multi-product SCND problem including suppliers, processing facilities, and warehouses. They also proposed an algorithm for solving the problem and they used it for two realistic SCD problems. Azaron et al. [4] developed a multi-objective stochastic programming approach for SCD under uncertainty. They considered numerous uncertain parameters such as: demands, supplies, processing, transportation, and capacity modification costs. The common points of these proposed approaches are: (i) minimizing cost or maximizing profit as a single objective is often the optimization focus. Literature surveys conducted by Seuring et al. [36] and Benjaafar et al. [5] have identified a growing need for developing quantitative models, methods and approaches in sustainable SCND, (ii) in most multi-objective SCND approaches, only demand is considered as the uncertainty source [16], (iii) in most SCND problems under uncertainty only a few scenarios can be considered in the optimization process due to the complexity of the problem. Although in most practical situations, the number of possible scenarios is large. According to [19] an importance based sampling approach must be developed to ensure that all important plausible facets are covered in the small sample of scenarios selected.

In literature, existing analytic hierarchy process approaches are applied to facilitate location problems, where logistic actors are not considered in the selection criteria [3, 44]. Tuzkaya et al. [43] included qualitative and quantitative criteria (benefits, opportunities, costs and risks), to assess and to select undesirable facility locations. Kinra and Kotzab [18] suggested a simple AHP model to include constraints such as government regulations, policy, infrastructural and political conditions, to find the best location of an industrial park. Few papers integrated ET in SC problems one of them is Wu and Barnes [48], which used the ET for formulating criteria to use in partner selection decisionmaking in agile supply chains. Wu [48] proposed a supplier selection in a fuzzy group setting involving decisions balancing a number of conflicting criteria and opinions from different experts. In addition, only a few works got interested in integrating data mining tools in the SC problems [8, 26] despite their various applications [22]. Data mining techniques can be used to improve strategic and operational planning activities [25]. However, SC literature lacks works that integrate imperfect data mining, which is an appropriate tool in case of treating uncertain and imprecise data. More especially, no evidential data mining approach has been applied to SCND problems.

3.   THE EVIDENCE THEORY

The ET (also called Belief Function Theory (BFT))[1] was initiated by the work of Dempster [11] on upper and lower probabilities. The development of the theory formalism is owed to Shafer [37]. Shafer has shown the benefits of ET to model uncertain knowledge. In addition, it allows knowledge combination obtained through various sources and it offers more flexibility than the probabilistic framework does. Shafer's book [37] has been followed by a large literature on interpretation, application, and computation [15,41]. For example, Smets [39, 41] introduced a non probabilistic interpretation of ET called Transferable Belief Model (TBM). In the remainder, we build our contribution based on the Transferable Belief Model (TBM).

The Basic Belief Assignment (BBA) *m* is a function defined on each subspace of the set of disjunctions of 2^Θ and taking values in [0, 1]. Θ is the of all possible exclusive and exhaustive answers (hypothesis) for the treated question and is called *frame of discernment*. It does not only represent all the confidence granted to each possible response for the

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[1] In this paper we conjugate the use of the two designations of the theory. The belief function theory designation is associated to the AHP method whereas ET is linked to evidential data mining.

treated question but also the ignorance and the lack of certitude. A BBA m is modelled as follows:

$$\begin{cases} \sum_{A \subseteq \theta} m(A) = 1 \\ m(\emptyset) \geq 0 \end{cases} \qquad (1)$$

Each hypothesis A having a belief value greater than 0 is called a *focal element*. $m(\emptyset)$ is called the *conflictual mass*. A BBA is called *normal* whenever the emptyset is not a focal element and this corresponds to a closed world assumption [38], otherwise it is said *subnormal* and it corresponds to an open world assumption [41]. The closed world assumption is the assumption that what is not known to be true must be false. The Open World Assumption is the opposite. For more details, an example of BBA modelling is shown in Annex A (Example 3).

When several information, modelled by BBAs, are induced from different experts, a fusion process is required. The ET proposes several operators of combination. One of the most used is the conjunctive rule of combination. This combination operator is used when sources (i.e. experts) are distinct and reliable. For two sources S_1 and S_2 having respectively m_1 and m_2 as BBAs, the conjunctive rule of combination is written as follows:

$$m_{\bigcirc}^{\Theta} = m_1^{\Theta} \bigcirc m_2^{\Theta}. \qquad (2)$$

For an event $A \neq \emptyset$, $m_{\bigcirc}^{\Theta}$ can be written as follows:

$$\begin{cases} m_{\bigcirc}^{\Theta}(A) = \sum_{B \cap C = A} m_1^{\Theta}(B) \cdot m_2^{\Theta}(C) \qquad \forall A \subseteq \Theta \\ m_{\bigcirc}^{\Theta}(\emptyset) \geq 0. \end{cases} \qquad (3)$$

where $m_{\bigcirc}^{\Theta}(\emptyset)$ corresponds to the conflict mass and highlights the degree of contradiction between the combined sources (for further details the reader can refer to Annex A, Example 4).

Decision functions allow the determination of the most suitable hypothesis from a BBA for the treated problem. In the TBM model, the pignistic level (i.e., decision level) allows to make decision from classical probabilities. The pignistic probability [40], denoted $BetP$, was introduced by Smets [40] within its TBM model. Not only does it makes probability transformation but it also takes into consideration the composite nature of focal elements. Formally, $BetP$ is defined as follows:

$$BetP(H_n) = \sum_{A \subseteq \Theta} \frac{|H_n \cap A|}{|A|} \cdot \frac{m(A)}{1 - m(\emptyset)} \qquad \forall H_n \in \Theta. \qquad (4)$$

An example of pignistic probability use is provided in Example 5 in Annex A.

In the following, we introduce the TSSP formulation to design a SCN answering a real-world problem.

4. SOLUTION APPROACH

This section presents a real-world SCND problem. The potential design of a supply chain being considered by a textile company in France (Fig. 1) is composed of suppliers, production centres, and customers. Several criteria should be integrated into the model, such as the total SC cost, jobless level, the quality of the location and the environmental aspect. These criteria may affect the selection of the facility locations, which is important for making an optimal decision. The company is only interested in the SCND problem from the strategic point of view and model it as a multi-level, single-product and single-period problem. Accordingly, the modelling assumptions are as follows:

1. production costs [35], transportation costs [4], demands [29] and supplied quantities [35] are uncertain parameters.

2. products are aggregated into a single product shipped through the SC network in a unique long-term period

In the following, we detail the steps of our approach. This approach, as illustrated in Fig. 2, has four steps. The first step consists in storing the experts' preferences over a set of criteria in order to process them with BF-AHP. In fact, the second step is related to the best locations selection i.e., where facilities can be set-up respecting all criteria: environmental, social, and economical criteria. The BF-AHP method [6, 12, 13], that provides a convenient framework for

dealing with uncertain data given by experts, is then used to select the best alternative. As highlighted in [14], classical AHP method is often criticized for its use of an unbalanced scale of estimations and its inability to adequately handle uncertainty and imprecision associated with the mapping of the decision maker's perception to a crisp number. In

Figure 1: Supply Chain Network
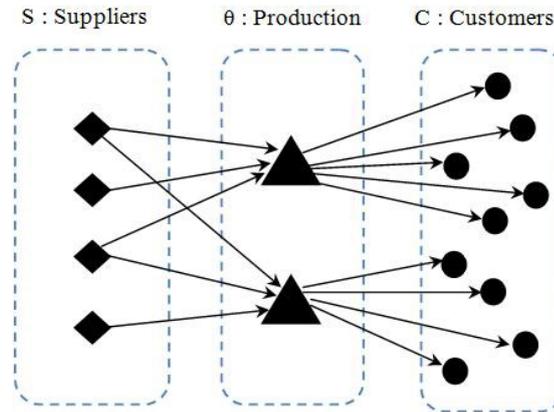
addition, BF-AHP tolerates that an expert may be uncertain about his level of preference due to incomplete information or knowledge, inherent complexity and uncertainty within the decision environment. Therefore, BF-AHP is an interesting approach to tackle uncertainty and imprecision within the pair-wise comparison process, in particular, when the decision maker's judgements are represented as a qualitative assessment. Once the best facilities are located, in the third step, experts are questioned over the optimal SCN configuration (called also scenarios) to meet customer demand minimizing the sum of costs. To select a subset of scenarios from a large set given by experts, for solving the TSSP model, the evidential data mining tools are used. Evidential data mining is very suited to handle large uncertain data and to associate a support value (degree of frequency of an item within the database) to each selected scenario. So, the last step combines ET and the TSSP model. These steps will be detailed in the following subsections.

Belief Function AHP

The introduction of the BFT to the AHP method has brought flexibility in data treatment. Indeed, imprecision and uncertainty are handled in the decisionmaking problem. The first step of this method consists in the identification of the set of criteria Ω and alternatives Θ. The expert can not only express his preferences on the selected criteria but he can also do it on a set of them. Once the sets of criteria and alternatives are defined, the expert tries to specify his preferences in order to obtain the criterion weights and the alternative scores in terms of each criterion. Interested readers may refer to [13,14] for more detail of pair-wise comparisons and to preference elicitation. At the alternative level, unlike the criterion level, the expert tries to express his preferences over the sets of alternatives regarding each criterion. Accordingly, and to better imitate the expert reasoning, we indicate that to define the influences of the criteria on the evaluation of alternatives; we might use conditional belief

Figure 2: Approach steps
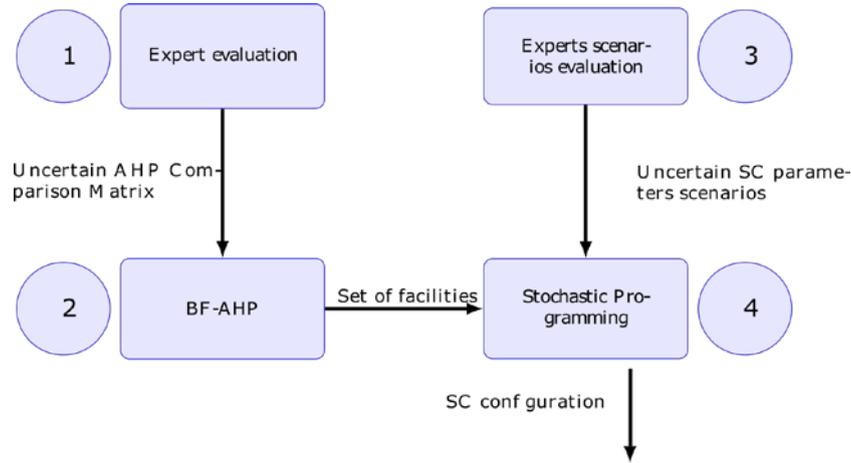
that could be written as follows:

$$m^{\Theta}[c_j](A_k) = \omega_k, \qquad \forall A_k \subset 2^{\Theta} \text{ and } c_j \in \Omega \tag{5}$$

where A_k represents a subset of 2^{Θ}, ω_k is the eigen value of the kth sets of alternatives regarding the criterion c_j. $m^{\Theta}[c_j](A_k)$ means that we know the belief about A_k regarding c_j. The BF-AHP aims through this step to combine the obtained conditional belief with the importance of their respective criteria to measure their contribution. Since the group of criteria and alternative that respectively belong to the frame of discernment Ω and Θ are different, the extension for constructed BBA is needed. At criterion level, the BBA m^{Ω}

is extended to $\Theta \times \Omega$ with the use of Equation (21)[2]. On the other hand, at alternative level, the ballooning extension [28] is applied with the use of Equation (23)[3]. The ballooning extension consists at expressing a BBA, initially defined in the frame of discernment Θ, in a higher set $\Theta \times \Omega$. Finally, we might combine the obtained BBA with the importance of their respective criteria to measure their contribution. To that purpose, we will apply the conjunctive rule of combination so that:

$$m^{\Theta \frown \Omega} = [Q \cap_{j=1,...,m} m^{\Theta}[c_j]^{"\,\Theta \frown \Omega}]Q \cap m^{\Omega\,'\,\Theta \frown \Omega} \tag{6}$$

where $m^{\Omega \uparrow \Theta \times \Omega}$ is the vacuous extension of m^{Ω} which reflects the importance of criteria. In order to make a decision, a marginalization[4] is operated on the resulting BBA over Θ with Equation (22). the decision is made over a set of probabilities found with the application of the pignistic probability (Equation (4).

4.2 Evidential data mining and stochastic programming

In this step, we address the problem of reducing the number of scenarios in TSSP. The location of the facilities already computed with BF-AHP, we aim now at finding the best scenarios (i.e., the best configuration). To do so, we access the opinion of several experts over set of parameters such as: the quantity (pieces), the transport unitary cost, the production unitary cost and the demand. To handle these imprecise data (opinions), an uncertain data mining approach (or commonly)

called uncertain pattern mining approach) is required. To deal with this problem, we propose the Expert Decision Consensus Approach (EDeCA) based on evidential data mining [17, 34], which reduces the dimensions of the original scenarios set to a smaller set of scenarios. The choice of evidential data mining is motivated by its ability to model expert's opinions even if they suffer from imprecision and uncertainty [21]. In addition evidential data mining provides a generalizing framework for binary and other imperfect data

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[2] Details about vacuous extension of Equation (21) are provided in the appendix.

[3] Details about operation in product space in belief function theory are provided in the appendix.

[4] Details about marginalization of Equation (22) are provided in the appendix.

mining toolboxes [33]. Then, the stochastic program is solved with the smaller set of scenarios in order to obtain a representative solution in reasonable time. In this subsection, we recall some basics of evidential data mining.

An evidential database is a database in which expert opinions are expressed with basic belief assignment functions. Let us consider the set of n experts $E = \{E1, \cdots, En\}$. Each expert Ej gives his opinion with regards to a parameter (e.g the cost) $i \in I$ that takes its value from the set Θi. The set $\Theta_i = \{\omega, \frac{1}{i}\omega\}$ is all possible choices ($k \in [1, 2^{|\Theta i|}]$) that any expert i   I and a parameter i, a BBA may pick for the parameter i. For an expert Ej and a parameter i, a BBA $m_j^{\Theta_i} : 2^{\Theta_i} \longrightarrow [0, 1]$ is modelled as follows:

$$\begin{cases} \sum_{\omega_i^k \subseteq \Theta_i} m_j^{\Theta_i}(\{\omega_i^k\}) = 1 \\ m_j^{\Theta_i}(\emptyset) = 0. \end{cases} \qquad (7)$$

The constructed evidential database that summarizes expert opinions is denoted EDB. A focal element is commonly called an item. An itemset is a set of focal elements belonging to different parameters. In this work, an itemset having a size I (itemset containing items from each parameter) represents a feasible scenario.

Example 1 Table 1 shows an example of two experts having their opinions captured in an evidential database. The cost parameter takes its values withinthe frame of discernment $\Theta_1 = \{High_1, Low_1\}$ whereas the quantity parameter are within $\Theta_2 = \{High_2, Low_2\}$. $m_i^\theta(\{w_i^k\})$ highlights how confident expert *j* is that the parameter should be ω_i^k.

Table 1: Supply chain parameter's evidential database *EDB*

Expert	Cost	Quantity
E_1	$m_1^{\Theta_1}(\{High_1\}) = 0.7$ $m_1^{\Theta_1}(\Theta_1) = 0.3$	$m_1^{\Theta_2}(\{High_2\}) = 0.4$ $m_1^{\Theta_2}(\{Low_2\}) = 0.2$ $m_1^{\Theta_2}(\Theta_2) = 0.4$
E_2	$m_2^{\Theta_1}(\{Low_1\}) = 0.3$ $m_2^{\Theta_1}(\Theta_1) = 0.7$	$m_2^{\Theta_2}(\{High_2\}) = 1$

The first row of the database means the expert *E*1 thinks that the cost should have a high value but he remains confused ($m_1^{\Theta 1}(\Theta 1) > 0$). Expert *E*2 is sure that the quantity should remain high (i.e, $m_2^{\Theta 2}(\{High_2\}) = 1$). $High_1$ is an item whereas $High_1 \times Low_2$ is a scenario.

Once the supply chain parameter's evidential database is constructed, the main task is to mine interesting scenarios. As it is the case for the other variant of data mining, the aim is to retrieve items and itemsets that have a degree of presence (aka support) within the database greater than or equal to a fixed threshold. In our problem, it means that we are aiming at retrieving valuable information supported by the majority of experts. In this part, we introduce a new measure that we denote as the *precise measure*. This measure computes the pertinence of all scenarios. For an item ω_i^k, the precise measure is computed as follows:

$$Pr : 2^{\Theta} \to [0, 1] \qquad (8)$$

$$Pr(\omega_i^k) = \sum_{\omega \subseteq \Theta_i} \frac{|\omega_i^k \cap \omega|}{|\omega|} \times m(\omega) \qquad \forall \omega_i^k \in 2^{\Theta_i}. \qquad (9)$$

As illustrated above, the Pr(.) is a measure that computes the probability of presence of an item in a single BBA. The Pr is also assimilated to the pignistic probability in case of $\omega_i^k \in \Theta_i$. For an itemset $X = \prod_{i \in [1...I]} \omega_i^k$ and satisfying the constraint

$\bigcap_{i \in I, k \in 2^{|\Theta_i|}} \omega_i^k = \emptyset$, its support (i.e., pertinence) for a single expert E_j is computed with the precise measure as follows:

$$Pr_{E_j}(X) = \prod_{\omega_i^k \in X} Pr(\omega_i^k). \qquad (10)$$

Finally, the support of X in the entire database is deduced as follows:

$$Pr_{\mathcal{EDB}}(X) = \frac{1}{n} \sum_{j=1}^{n} Pr_{E_j}(X) \qquad (11)$$

Example 2 Let us consider the Table 1. The precise measure of the scenarios $X1=\{High_c, Low_q\}$ and $X2 = \{High_c, High_q\}$ is computed as follows:

$$Pr_{\mathcal{EDB}}(X_1) = \frac{Pr_{E_1}(X_1) + Pr_{E_2}(X_1)}{2} = \frac{0.85 \times 0.4 + 0}{2} = 0.17$$

$$Pr_{\mathcal{EDB}}(X_2) = \frac{Pr_{E_1}(X_2) + Pr_{E_2}(X_2)}{2} = \frac{0.85 \times 0.6 + 0.35 \times 1}{2} = 0.43$$

The precise measure aims at estimating the support of each itemset. In our problem, an itemset is assimilated to a scenario. The more the support grows for a scenario, the more priority it gains. supported by the majority of experts. The scenarios having the highest precise value (i.e., support) gather somehow the consensus of studied experts. Therefore, the approach of extracting pertinent scenarios with the precise measure is denoted as *Expert Decision Consensus Approach* (EDeCA). In addition, the precise support provides very precise estimation of the support comparatively to the other state-of-the-art alternative which is the belief support [17]. As demonstrated recently in [31, 34], the belief support provides a pessimistic estimation of the support and therefore do not outperform the precise support.

In pattern mining the definition of support is related to a threshold called *minsup* [1]. Let us assume a threshold *minsup* and a scenario X, X is considered as *frequent* as long as $Pr_{EDB}(X) \geq minsup$. The threshold limits the number of retained scenarios. Only scenarios with a support greater or equal to *minsup* are retained in the set of frequent scenarios FS. The threshold is generally fixed by the expert. However, it still can be automatically computed to select only pertinent scenarios [27]. In Example 2, for a *minsup* fixed to 0.4, X_2 is considered as a frequent scenario.

Proposition 1 *Let us consider the database EDB of n experts expressing their opinions over I parameters. Let us consider two scenarios X_1 and X_2. X_1 has more priority than X_2 if and only if $Pr_{EDB}(X_1) > Pr_{EDB}(X_2)$.*

Despite, the frequency threshold constraint, the number of frequent scenarios could still be to great to be evaluated. Therefore, several approaches for selecting only the *top-k* best scenarios were provided [47]. A top-*k* mining approach consists in retaining only the *k* frequent scenarios with the highest utility. In this work, we fix *k* parameter proportionally to the size of the database such as $k = \lceil \frac{n*500}{100} \rceil$ (five percent of the database). Therefore, the number of retrieved pertinent scenario do never exceed 5% of the database size. For the utility function, it is derived from Proposition 1 where only those frequent scenarios maximizing the support are retained.

Using the set of facility locations obtained solving the belief AHP and the reduced set of scenarios along with their precise measures obtained by solving EDeCA approach, we design the supply chain network of the problem solving the TSSP presented in Equations (12) to (19).

We consider a network *G=(NO, AC) (Fig. 1),* where *NO* is the set of nodes and *AC* is the set of arcs. The SCN consists of the set of suppliers S, the set of processing facilities *Θ* and the set of customers *C*. *R* is the set of scenarios. The SC configuration decisions consist in deciding which of the manufacturing facilities to open and the quantity of products transported through the SCN. We associate a binary variable y_i to the location decisions, $y_i=1$, if a manufacturing facility i is built, and 0 otherwise. We let x_{ij} denote the flow of product from a node i to a node j of the network where (ij) ∈ AC.

We model the SC as two-stage stochastic program (see [2, 4, 35, 45, 46]). For reasons of simplicity, we denote the set of feasible solution of first-stage decisions y_i by Y. The uncertain parameter in this formulation is: production costs, transportation costs, opening costs, demands and supplies. The first stage consists in determining the configuration decisions y, and the second stage consists of the quantities of goods to transport throughout the supply chain network in an optimal way. Note that ζ represents the random vector corresponding to the uncertain parameters. The design objective is to minimize the sum of investment costs and expected future variable costs. The strategic SCN that we intend to establish, should answer the following questions under uncertain environment: *(i)* how many manufacturing plants should be installed *(ii)* where the new sites should be located

(iii) how much goods the production plant should handle *(iv)* what products quantities to transport throughout the supply chain network. The minimization of the sum of investment can be written as follows:

$$Min \quad \sum_{i \in \overline{\Theta}} oc_i \cdot y_i + \sum_{s \in R} \rho_s Q(y, \xi^s) \qquad (12)$$

subject to:

$$y \in Y \subseteq \{0, 1^n\} \qquad (13)$$

with Q(y, ζ^s) being the solution of the following second stage problem:

$$Min \quad \sum_{i \in S, j \in \overline{\Theta}} \mu^s \beta_{ij} \cdot x_{ij}^s + \sum_{j \in \overline{\Theta}, k \in C} \mu^s \beta_{jk} \cdot x_{jk}^s + \sum_{i \in S, j \in \overline{\Theta}} f_j^s \cdot x_{ij}^s \, s \in R. \qquad (14)$$

subject to:

$$\sum_{i \in S, j \in \overline{\Theta}} x_{ij}^s \leq A_i^s \qquad i \in S; s \in R. \qquad (15)$$

$$\sum_{i \in S} x_{ij}^s - \sum_{k \in C} x_{jk}^s = 0 \qquad j \in \overline{\Theta}; s \in R. \qquad (16)$$

$$\sum_{i \in S} x_{ij}^s \leq F_j^{max} \cdot y_j \qquad j \in \overline{\Theta}; s \in R. \qquad (17)$$

$$\sum_{j \in \overline{\Theta}} x_{jk}^s \geq d_k^s \qquad k \in C, s \in R. \qquad (18)$$

$$x_{ij}^s \geq 0 \qquad i \in S, j \in \overline{\Theta}, s \in R. \qquad (19)$$

$$x_{jk}^s \geq 0 \qquad j \in \overline{\Theta}, k \in C, s \in R. \qquad (20)$$

where oc_i denotes the opening cost for building facility i, and ρs is the proba bility of each scenario. The first-stage consists of configuration decisions y, the objective is to minimize investment costs and future operating costs (Equation 12). The second-stage consists of optimal production and transportation of products through the SC based on the configuration and the realized uncertain scenario (Equation 14). f_j^s is the unit production cost of each product at facility j and μs presents unit transportation cost of each product and βij is the distance between i and j on arc (*ij*). Constraint (15) requires that the total flow of products from a supplier j, should be less than the supply $_{Aj}$ at that node. Constraint (16) enforces the flow conservation of products across each processing node j. Constraint (17) represents the capacity constraint who requires

that the total processing requirement of all products flowing into a processing node j should be smaller than the capacity F_j^{max} of facility j if it is built (y_j=1). If facility j is not built (y_j=0) the constraint will force all flow variables x_{ij}=0 for all $i \in N$. Constraint (18) requires that the total flow of products sent to a customer j should exceed the demand d_k of customer k. Finally, constraints (19) and (20) enforce the non-negativity of the flow variables.

5. COMPUTATIONAL EXPERIMENTS

In this section, we present the results of the analysis. In order to evaluate the model, we present an example of SC. The SCN consists of 4 possible locations for production center, 3 suppliers from

whom goods are supplied, and 10 customers that the company serves. We propose an ET-TSSP model in which we involve 100 experts' scenarios and 4 criteria: investment costs, environmental risks, location, social aspect. The uncertain parameters are: the experts decisions in the selection of the best facility location set, customer demand, transportation costs, production costs and supplied quantities. One hundred scenarios are proposed by several experts for TSSP and all these scenarios are reduced to 5 best scenarios using the evidential data mining tool. The TSSP problem was solved using ILOG CPLEX 12.0 solver. All the experiments are conducted on a PC with Intel Core 2 Duo 2.19 GHz and 2 GB RAM.

A comparison between the deterministic approach and the stochastic solution is performed next.

5.1 Belief AHP results

Before running the TSSP model, the first step is to obtain a set of potential facility locations. As mentioned in section 4, the potential set of facilities needs to be computed using BF-AHP. To select the best facility locations, we relied on four important criteria: $\Omega=\{$investment costs (C_1), environmental risks (C_2), location (Accessibility, closeness to suppliers and customers) (C_3), social aspect $(C_4)\}$.

We have to select the best locations from four location alternatives: $\Theta=\{L_1, L_2, L_3, L_4\}$ as shown in Figure 3:
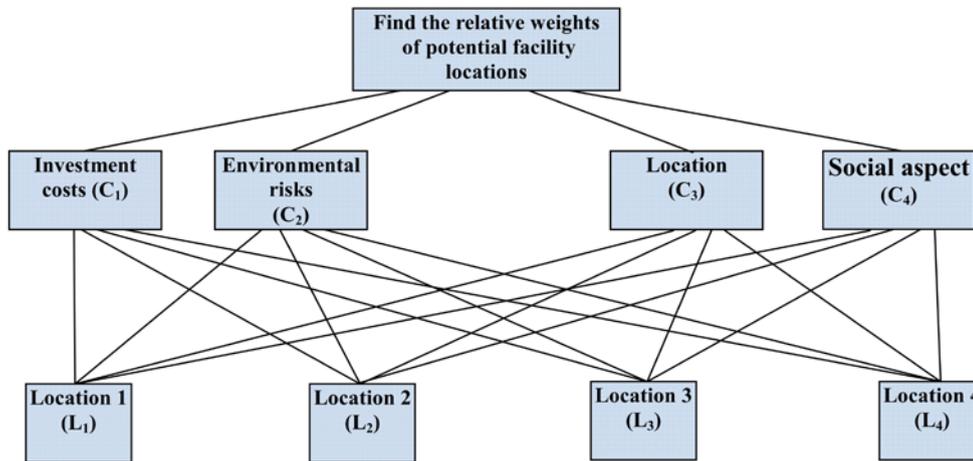
Figure 3: AHP hierarchy: Find the best facility location.

Table 2 shows the weight assigned to each criterion given by an expert of the textile company. These weights are then converted into a BBA m^{Ω}.

Table 2: Weights assigned to criteria according to the experts'

opinion Criteria		$\{C_1\}$	$\{C_2\}$	$\{C_3, C_4\}$
	Priority			
$\{C_1\}$	1	3	$\frac{1}{}$	0.31
$\{C_2\}$	$\frac{1}{}$	1	5	0.35
$\{C_3, C_4\}$	4	$\frac{1}{}$	1	0.34

In the same way as criterion level, the expert is asked to evaluate subsets of alternatives according to each criterion C_i. With these comparison judgement matrix, the priority values are obtained in regard to each criterion (Table 3).

Table 3: Priority values

$\{C_1\}$	Priority	$\{C_2\}$	Priority	$\{C_3\}$	Priority	$\{C_4\}$	Priority
L_1	0.28	L_3	0.73	L_1	0.24	L_3	0.35
$\{L_1 \cup L_3\}$	0.33	$\{L_1 \cup L_2 \cup L_4\}$	0.27	L_2	0.33	Θ	0.65
L_4	0.19			Θ	0.43		
Θ	0.20						

Table 4 summarizes the ballooning extension operated on a conditional alternative BBA according to C_1. Indeed, from priority values, an extension is operated following Equation (23). This operation is repeated for the other priority values obtained according to criteria.

Table 4: Ballooning extension $m^\Theta [C1]^{\Uparrow\Theta\times\Omega}$ of conditional BBA $m^\Theta [C1]$

conditional bbm	Ballooning extension	Values
$m^\Theta[C_1](\{L_1\})$	$\{(L_1, C_1), \Theta \times \{C_2, C_3, C_4\}\}$	0.28
$m^\Theta[C_1](\{L_2 \cup L_3\})$	$\{(L_2, C_1), (L_3, C_1), \Theta \times \{C_2, C_3, C_4\}\}$	0.33
$m^\Theta[C_1](\{L_4\})$	$\{(L_4, C_1), \Theta \times \{C_2, C_3, C_4\}\}$	0.19
$m^\Theta[C_1](\Theta)$	$\Theta \times \Omega$	0.20

The final results are shown in Table 5 in which the resulting BBA is detailed. This BBA is obtained, with Equation (6) after combining extended conditional BBAs ($m^\Omega [C_i]^{\Uparrow\Theta\times\Omega}$) with criteria BBA ($m^\Omega$) which is previously extended to $\Theta\times\Omega$ and marginalization on Θ (Equation (22)).

Table 5: The obtained BBA $m^{\Theta\times\Omega\downarrow\Theta}$ and the confidence of location possibilities

$\{L_1\}$	0.13
$\{L_2\}$	0.07
$\{L_3\}$	0.30
$\{L_4\}$	0.06
$\{L_1, L_3\}$	0.04
$\{L_2, L_3\}$	0.14
$\{L_1, L_2, L_4\}$	0.10
Θ	0.09
$\varnothing$	0.07

After the pignistic transformation, the results obtained from the computation of the weights using the BF-AHP method for four criteria and four alternatives are shown in Table 6. The priority weights of the four alternatives (L_1, L_2, L_3, L_4) are 0.22, 0.21, 0.45, 0.12 respectively. They imply that the third alternative (L_3) and the first alternative (L_1) are the best facility locations. These locations are then used in the second step to design the SCN using TSSP.

Table 6: The final BBA m^Θ and locations' priority

Alternative	priority
L_1	0.22
L_2	0.21
L_3	0.45
L_4	0.12

5.2 EDeCA results

The tests were conducted in a database that contains 100 records. A part of the records are collected from experts of the textile company. This study case was also presented to several scientists. All experts were allowed to give more than one opinion if they desire.

Each expert gives values over four parameters: quantity A_i (pieces), transport unitary cost (€/pcs.km), production unitary cost (€/pc) and demand (pcs). These informations are depicted in Table 7.

Table 7: Expert's chosen parameters database

Expert	Quantity A_i (pieces)	Transport unitary cost (€/pcs.km)	Production unitary cost (€/pc)	Demand (pcs)
Expert 1	2381409	0.26	5.05	1148838
Expert 2	2399271	0.33	7.04	1077142
…	…	…	…	…
Expert 100	2046663	0.2	5.2	1060153
Mean	2222917	0.3	7.64	1259580
Min	2002113	0.11	5.01	1001389
Max	2499212	0.50	9.89	1497426
Median	2214462	0.3	7.73	1250971
Standard deviation	147200	0.12	1.48	145745

In order to obtain its corresponding evidential database, an evidentialization process [32] is operated using ECM algorithm [23]. The resulting evidential database is illustrated in Table 8.

Table 8: Expert's chosen parameters evidential database

Expert	BBA Quantity $\Theta_1 = \{\omega_1^1, \omega_1^2, \omega_1^3, \omega_1^4, \omega_1^5\}$	BBA Transport $\Theta_2 = \{\omega_2^1, \omega_2^2, \omega_2^3\}$	BBA Production $\Theta_3 = \{\omega_3^1, \omega_3^2, \omega_3^3, \omega_3^4, \omega_3^5\}$	BBA Demand $\Theta_4 = \{\omega_4^1, \omega_4^2, \omega_4^3, \omega_4^4, \omega_4^5\}$
Expert 1	$m_1^{\Theta_1}(\{\omega_1^2\}) = 0.02$ $m_1^{\Theta_1}(\omega_1^2 \cup \omega_1^4) = 0.07$ $m_1^{\Theta_1}(\omega_1^3 \cup \omega_1^4) = 0.89$ $m_1^{\Theta_1}(\Theta_1) = E$	$m_1^{\Theta_2}(\{\omega_2^3\}) = 0.021$ $m_1^{\Theta_2}(\omega_2^1 \cup \omega_2^2 \cup \omega_2^3) = 0.82$ $m_1^{\Theta_2}(\{\omega_2^4\}) = 0.11$ $m_1^{\Theta_2}(\Theta_2) = E$	$m_1^{\Theta_3}(\{\omega_3^2\}) = 0.85$ $m_1^{\Theta_3}(\omega_3^1 \cup \omega_3^3) = 0.04$ $m_1^{\Theta_3}(\{\omega_3^3\}) = 0.05$ $m_1^{\Theta_3}(\Theta_3) = E$	$m_1^{\Theta_4}(\omega_4^1 \cup \omega_4^5) = 0.94$ $m_1^{\Theta_4}(\omega_4^2 \cup \omega_4^3) = 0.01$ $m_1^{\Theta_4}(\Theta_4) = E$
…	…	…	…	…
Expert 100	$m_{100}^{\Theta_1}(\{\omega_1^3\}) = 0.07$ $m_{100}^{\Theta_1}(\{\omega_1^5\}) = 0.03$ $m_{100}^{\Theta_1}(\omega_1^1 \cup \omega_1^5) = 0.86$ $m_{100}^{\Theta_1}(\Theta_1) = E$	$m_{100}^{\Theta_2}(\{\omega_2^1\}) = 0.04$ $m_{100}^{\Theta_2}(\omega_2^2 \cup \omega_2^3) = 0.09$ $m_{100}^{\Theta_2}(\Theta_2) = 0.87$	$m_{100}^{\Theta_3}(\{\omega_3^1\}) = 0.99$ $m_{100}^{\Theta_3}(\Theta_3) = E$	$m_{100}^{\Theta_4}(\{\omega_4^4\}) = 0.16$ $m_{100}^{\Theta_4}(\omega_4^1 \cup \omega_4^4) = 0.07$ $m_{100}^{\Theta_4}(\omega_4^4 \cup \omega_4^5) = 0.57$ $m_{100}^{\Theta_4}(\Theta_4) = E$

Since the evidentialization relies on the ECM [23] approach, Fig. 4 details same approach is applied for the parameter Transport unitary cost, Production unitary cost and demands. The results of these evidentialization process are presented in Fig. 5, Fig. 6 and Fig. 7 respectively.

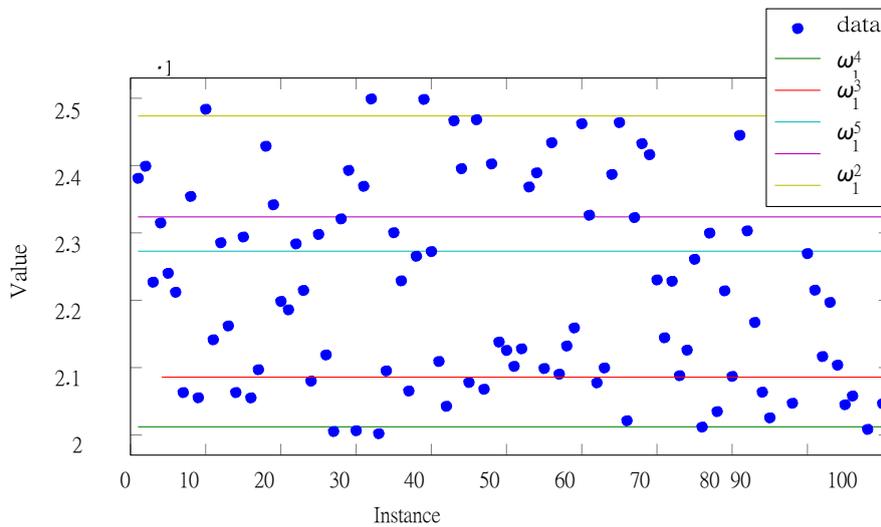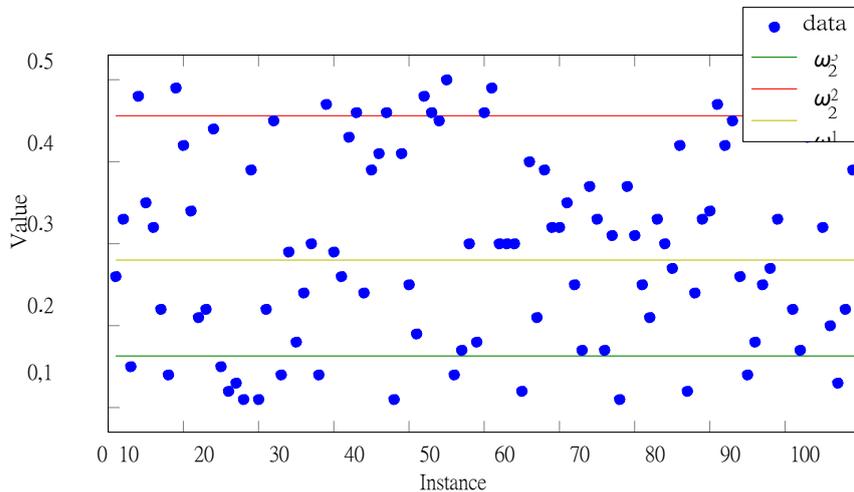Figure 4: Cluster's centers for the Quantity parameter

Figure 5: Cluster's centers for the Transport unitary cost

The results of the EDeCA approach are illustrated in Table 9 and 10. EDeCA retrieved over 900 frequent scenarios from which we distinguish two categories. The first category regroups over 500 imprecise scenarios which we denote as the vague ones. Indeed, to each single parameter, the EDeCA approach retains a vague focal element (disjunction of hypothesis). These kinds of scenario are important since it gives the decision maker an idea about the values that he must avoid and those he must consider the top-k best scenarios and they.

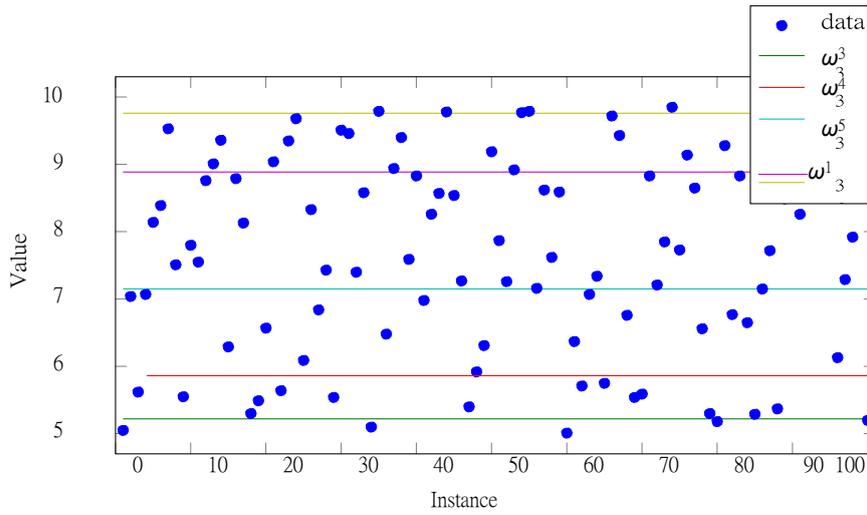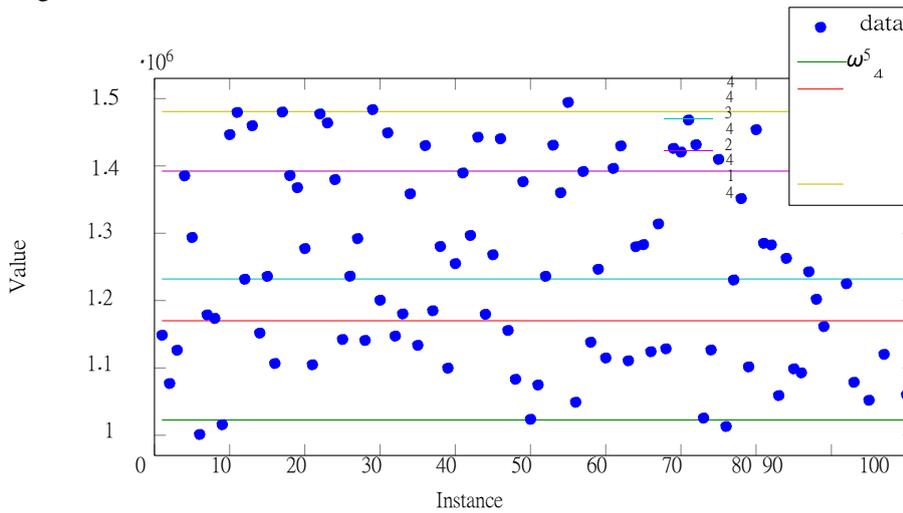Figure 6: Cluster's centers for the Production unitary cost

Fig. 7 Cluster's centers for the Demands

are shown in Table 9 where k is fixed to 5. So, these scenarios are then ranked by their pertinence. The second category regroups the precise frequent scenarios i.e., those constituted by simple focal elements (singleton hypothesis). The precise scenarios point out which parameter's value the decider has to choose. The top-5 precise scenarios are shown in Table 10.

Table 9: 5 best vague scenarios

Scenario	Pertinence
$\{\omega^1 \cup \omega^2 \cup \omega^3 \cup \omega^4\}, \Theta_2, \{\omega^1 \cup \omega^2 \cup \omega^3 \cup \omega^4\}, \Theta_4$	0.773
$\{\omega_1^1 \cup \omega_1^2 \cup \omega_1^3 \cup \omega_1^4\}, \Theta_2, \{\omega_3^1 \cup \omega_3^2 \cup \omega_3^3 \cup \omega_3^5\}, \Theta_4$	0.712
$\{\omega_1^1 \cup \omega_1^3 \cup \omega_1^4 \cup \omega_1^5\}, \Theta_2, \{\omega_3^1 \cup \omega_3^2 \cup \omega_3^3 \cup \omega_3^4\}, \Theta_4$	0.709
$\{\omega_1^1 \cup \omega_1^3 \cup \omega_1^4 \cup \omega_1^5\}, \Theta_2, \{\omega_3^1 \cup \omega_3^2 \cup \omega_3^3 \cup \omega_3^5\}, \Theta_4$	0.699
$\{\omega_1^1 \cup \omega_1^2 \cup \omega_1^3 \cup \omega_1^4\}, \Theta_2, \{\omega_3^1 \cup \omega_3^2 \cup \omega_3^3 \cup \omega_3^5\}, \Theta_4$	0.689

Table 10: 5 best precise scenarios

Scenario	Pertinence	Probability of scenario
$\omega_1^3, \omega_2^3, \omega_3^2, \omega_4^1$	0.015	0.28
$\omega_1^4, \omega_2^3, \omega_3^2, \omega_4^1$	0.012	0.22
$\omega_1^4, \omega_2^3, \omega_3^2, \omega_4^1$	0.009	0.17
$\omega_1^4, \omega_2^3, \omega_3^2, \omega_4^1$	0.009	0.17
$\omega_1^4, \omega_2^2, \omega_3^2, \omega_4^1$	0.009	0.16

By comparing the pertinence in Table 9 and 10, we notice that the vague scenarios present a better pertinence than the precise ones. This result was to be expected due to the nature of the pertinence formula (Equation (11)) and its pignistic probability basics. The more the considered scenario is vague, the more pertinence it gathers. On the other hand, our EDeCA approach gives a classification of both scenario categories. Even if the pertinence of the precise scenarios is low, EDeCA shows a ranked list of the best scenarios to rely on.
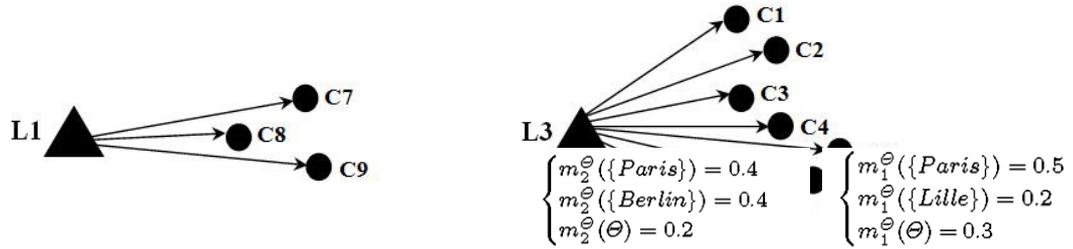
5.3 Two-stage stochastic model results

In this step, we study the configuration of the supply chain network. By using the set of facility locations and the reduced set of scenarios obtained in the preview step we solve the mathematical model (12)-(19). The values for the SC uncertain parameters are the same as in Table 10. In order to generate a balanced network configuration between these various scenarios, we applied stochastic programming with equal probabilities. Table 11 summarizes the results of the TSSP model, deterministic (DET), and normal distribution model (NDM). TSSP solution is obtained solving the TSSP model considering the 5 scenarios of EDeCA results and the best two facility locations L_1 and L_3.

TSSP4 solution is obtained solving the TSSP model considering the 5 scenarios of EDeCA results and the four location alternatives: $\Theta = \{L_1, L_2, L_3, L_4\}$.

A deterministic model is used to solve each scenario individually $(S_1,..,S_5)$. Scenarios $(S_6,..,S_{10})$ are generated assuming that the uncertain parameters fit to normal distribution.

Table 11 reveals that, for this case, the TSSP model contains 148 variables and 235 constraints. As we can see, for 5 scenarios the CPU Time is equal to 195 seconds, and it can easily go up with the growing of scenario numbers and the size of the supply chain. The SC configuration proposed is to open two production plants L_1 and L_3. The affectation of customers to each production facility is depicted in Fig. 8. To satisfy customers demand the production plant L_1 should deliver products to 3 customers (C_7, C_8, C_9) and L_3 should supply products to customers ($C_1, C_2, C_3, C_4, C_5, C_6, C_{10}$). The solution of TSSP4 is to open two production facilities L_3 and L_4. The production plant L_3 delivers the products to four customers (C_1, C_2, C_3, C_5) and L_4 supplies the other customers. The best facilities selected in this case are different than the facilities obtained solving the BF-AHP, because only the economic aspect was considered in the TSSP model. The solutions obtained using the TSSP model are feasible only because of the high capacity of the suppliers and the nonconsideration of the capacity constraints. Comparing the TSSP solution to the deterministic ones the structure of the SCN is the same in the all scenarios. This can be explained by the low uncertainty of the SC parameters and the small size of the case study. The NDM model exhibits two different SCNs, for scenarios (S_6,S_7,S_8,S_9) the optimal solution is to open two facilities L_1 and L_3 and for scenario S_{10}, the SC configuration proposed is to open one production plant (L_1).

Figure. 8: Customers affectation

$$\begin{cases} m_2^{\Theta}(\{Paris\}) = 0.4 \\ m_2^{\Theta}(\{Berlin\}) = 0.4 \\ m_2^{\Theta}(\Theta) = 0.2 \end{cases} \qquad \begin{cases} m_1^{\Theta}(\{Paris\}) = 0.5 \\ m_1^{\Theta}(\{Lille\}) = 0.2 \\ m_1^{\Theta}(\Theta) = 0.3 \end{cases}$$

6. CONCLUSIONS

In this paper, we introduce a multi-criteria SCND under uncertainty model based on the evidence theory. The main contribution of the model is the integration of multi-criteria aspect and uncertainty of SC parameters in the design of SCN using the belief AHP and TSSP.

The approach contains two steps, the aim of the first step is to select the best locations where plants can be opened. We used the belief AHP method to integrate uncertain information given by experts and to consider many criteria in the selection: environmental, social, and economical. In the second step, we consider that all SC parameters of the model are uncertain: transportation costs, production costs, customers demand and supplied quantities. So, we used the evidential data mining to select a subset of scenarios from a large set given by experts and TSSP to model the problem.

Several possible future research avenues can be defined in this context. For instance, addressing uncertainty in the suppliers capacity, the production capacity and the location of customers may be attractive direction for future research. Also, testing the approach on large scale SCN is not addressed in this paper. Therefore the evaluation of the model on large SCN and comparing its efficiency to other methods can be an interesting development in this area

Table 11 Computational Results

Method	Scenario	Variables	Constraints	CPU Time (s)	Optimal cost (€)	Production facility	L_1 clients	L_3 clients
TSSP	S1,..,S5	148	235	195	89 161 487	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
TSSP4	S1,..,S5	289	375	200	81 571 355	L_3,L_4	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
DET	S1	32	43	50	67 860 466	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
DET	S2	32	43	50	108 421 318	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
DET	S3	32	43	50	123 439 795	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
DET	S4	32	43	50	57 171 357	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
DET	S5	32	43	50	65 455 307	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
NDM	S6	32	43	50	97 562 178	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
NDM	S7	32	43	50	68 678 929	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
NDM	S8	32	43	50	103 739 889	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
NDM	S9	32	43	50	29 654 792	L_1,L_3	C7, C8, C9	C1, C2, C3, C4, C5, C6, C10
NDM	S10	32	43	50	50 349 427	L_1	-	C1, C2, C3, C4, C5, C6, C7, C8, C9, C10

A EXAMPLES ON BELIEF FUNCTION OPERATIONS DESCRIBED IN SECTION

Example 3 Let us assume a company that plans to relocate its factory to optimize its revenues. The factory should be either set up in the downtown of a big city or in its suburbs for supplying transport constraints i.e., Ω = {Downtown, Suburb}. Thus, three locations have emerged and are discussed i.e., $\Theta = \{Paris, Lille, Berlin\}$. Both Ω and Θ are frames of discernment. One expert has been questioned about the location problem and below is his answer modelled with a BBA.

$$\begin{cases} m^{\Theta}(\{Paris\}) = 0.5 \\ m^{\Theta}(\{Lille\}) = 0.2 \\ m^{\Theta}(\Theta) = 0.3 \end{cases}$$

$m^{\Theta}(\Theta)=0.3$ means that the expert has some doubts over the given location possibilities. This uncertainty is expressed by assigning a value to the frame of discernment Θ. .

Example 4 Let us consider Θ={Paris; Lille; Berlin}. Two experts have been questioned over the best possible location for the factory. Both opinions are highlighted in the following BBAs:

Thus, the result of the combination sum is equal to

$$\begin{cases} m_{\bigcirc}(\emptyset) = 0.36 \\ m_{\bigcirc}(\{Paris\}) = 0.42 \\ m_{\bigcirc}(\{Lille\}) = 0.04 \\ m_{\bigcirc}(\{Berlin\}) = 0.12 \\ m_{\bigcirc}(\Theta) = 0.06 \end{cases}$$

For example $m_1^\theta(\{Paris\}) x\, m_2^\theta(\{Paris\}) + m_1^\theta(\{Paris\})\, x\, m_2^\theta(\Theta) + m_2^\theta(\{Paris\})\, x\, m_1^\theta(\Theta)$

Example 5 Assuming the BBA obtained through the conjunctive rule of combination in Example 4. To make a final decision, it is possible to recover a set of probabilities from a BBA with the pignistic probability as follows:

$$\begin{cases} BetP(Paris) = 0.69 \\ BetP(Lille) = 0.09 \\ BetP(Berlin) = 0.22 \end{cases}$$

B Operation on the product space in belief function theory

Let $U = \{X, Y, Z, \ldots\}$ be a set of variables, each one has its frame of discernment. Let X and Y be two disjoint subsets of U . Their frames are the product space of the frames of the variables they include. Given a BBA defined on X, its vacuous extension on $X \times Y$ denoted $m^{X \uparrow X \times Y}$ is given by:

$$m^{X \uparrow X \times Y}(B) = \begin{cases} m^X(A)\, if\, B = A \times Y, A \\ \subset X, 0\, otherwise. \end{cases} \tag{21}$$

Example 6 Let us assume the example depicted in Example 3. The BBA defined on Θ will be defined in a finer frame $\Theta \times \Omega$ using the vacuous extension as follows:

$$\begin{cases} m^{\Theta \uparrow \Theta \times \Omega}(\{Paris, Downtown\}, \{Paris, Suburb\}) = 0.5 \\ m^{\Theta \uparrow \Theta \times \Omega}(\{Lille, Downtown\}, \{Lille, Suburb\}) = 0.2 \\ m^{\Theta \uparrow \Theta \times \Omega}(\Theta \times \Omega) = 0.3 \end{cases}$$

A BBA defined on a product space $X \times Y$ may be marginalized on X by transferring each mass $mX \times Y$

(B) from $B \subset X \times Y$ to its projection on X:

$$m^{X \times Y \downarrow X}(A) = \sum_{B \subset X \times Y | Proj(B \downarrow X) = A} m^{X \times Y}(B), \forall A \subset X \tag{22}$$

where Proj(B↓X) denotes the projection of B onto X.

Example 7 Let us assume the following BBA defined over $\Theta \times \Omega$:

$$\begin{cases} m^{\Theta \times \Omega}(\{Paris, Downtown\}, \{Paris, Suburb\}) = 0.5 \\ m^{\Theta \times \Omega}(\{Lille, Downtown\}, \{Lille, Suburb\}) = 0.2 \\ m^{\Theta \times \Omega}(\{Paris, Downtown\}) = 0.3 \end{cases}$$

Marginalizing of $m^{\Theta \times \Omega}$ on the coarser frame Θ gives the following $m^{\Theta \times \Omega \downarrow \Theta}$:

$$\begin{cases} m^{\Theta \times \Omega \downarrow \Theta}(\{Paris\}) = 0.5 + 0.3 \\ = 0.8\; m^{\Theta \times \Omega \downarrow \Theta}(\{Lille\}) = 0.2 \end{cases}$$

Let $m^X[B]$ represent the beliefs X conditionally on B a subset of Y , i.e., in a context where B holds. The balloning extension is defined as:

$$m^X[B]^{\Uparrow X \times Y}(A \times B \cup X \times B) = \overline{m}^X[B](A), \ \forall A \subset X. \tag{23}$$

Example 8 Let us consider $\Theta=\{Paris, Lille, Berlin\}$, $\Omega = \{Downtown, Suburb\}$ and the conditional BBA $m^\Theta[Downtown](\{Paris\})=0.6$. Its corresponding BBA on $\Theta \times \Omega$ is obtained by taking into consideration *{Paris, Downtown}* and all the instances of Θ for the complement of *{Downtown}.*

$m^\Theta[Downtown]^{\Uparrow \Theta \times \Omega}(\{(Paris, \ Downtown), \ (Paris, \ Suburb), \ (Lille, \ Suburb),(Berlin, \ Suburb)\}) = m^\Theta$ [*Downtown*](*{Paris}*).

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